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Some algebraic and topological structures of Fourier transformable functions

Dawood Khan[a], Homan Emadifar[b*], Ali Ahmed[a], Saleem Iqbal[a] and Muhammad Tariq [c]

[a]Department of Mathematics, University of Balochistan
Quetta, 87300 Pakistan
[b]Department of Mathematics, Hamedan Branch, Islamic Azad University
Hamedan, Iran
[c]Department of Statistics, Mehran University of Engineering and Technology
Jamshoro, Pakistan

E-mail: dawooddawood601@gmail.com
E-mail: homan_emadi@yahoo.com
E-mail: alihmd87@gmail.com
E-mail: saleemjqbal81@outlook.com
E-mail: captaintariq2187@gmail.com

Abstract. In this work, the set of all functions that are Fourier transformable with regard to their structure both algebraic and topological is taken into account. Certain topological properties of the set of Fourier transformable functions with the help of a metric are described. Also determines the proofs of the statements that the set of all Fourier transformable functions is a commutative semigroup with respect to the convolution operation as well as Abelian group with respect to the operation of addition. Metric for two functions belonging to the set of all Fourier transformable functions is defined and the proof that the Fourier transformable functions space is complete with our metric is given. The separability theorem and that the Fourier transformable functions space is disconnected are also discussed.

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*Corresponding Author
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Connected space.

1. **Introduction**

The Fourier transform was first initiated in 1807 and is named after Jean Baptiste Joseph Fourier. Fourier transforms are most well-known and well-established techniques in the areas of engineering and mathematics. Fourier transform technique characterizes the variable as a sum of complex exponentials. Fourier analysis has been utilized in digital image processing and signal processing for the anatomization of a single image as a 2D wave form, plus several other kind of form in particular Image Processing, Signal processing and Quantum mechanics. Fourier analysis also signifies filters, representation, encoding and Transformation, Data Processing and various other fields. Now a day the usage of Fourier transform in different applications has magnificated. Among the various transformation methods used in mathematics, this is one of the simplest due to which this method consumes less time. This transform is extensively used in mechanical system, power distribution system, wireless networks and industries. Generally in power distribution system, this transform is a quick, accurate, and noise-resistant method for modifying power quality disturbances. Furthermore it has vast area of its applications in cell phone and the use of it in the medical sciences. In cell phone, this transform utilizes the signal processing forms and the making of the mobile phone. The Fourier transform is an important tool for solving linear constant coefficient, ordinary or partial differential equations under appropriate initial and boundary value problems in the modern world. It is mainly a linear operator that transforms a function f(x) into a function F(&) with a complex argument & . The Fourier transform is a transformation from the time domain to the frequency domain used in the study of linear time-invariant systems (electrical circuits, harmonic oscillators, mechanical systems and optical devices). It is mostly related to Laplace transform. However the difference literally lies in that the Laplace transform patch up a function into its moments whereas Fourier transform expresses a function or signal as a series of modes of vibration. For further details (see the references ([1] -[13]).

Definition 1.1. *(Fourier Transform)*

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\&x} dx, (\& > 0) \tag{1.1}$$

Definition 1.2. *(Inverse Fourier Transform)*

$$F^{-1}\{f(\&)\} = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\&) e^{i\&x} d\&, (\& > 0) \tag{1.2}$$

Definition 1.3. *(Convolution of Fourier Transform) The convolution of $f_1(x)$ and $f_2(x)$ where $f_1(x)$ and $f_2(x)$ are the Fourier transformable functions is represented by $f_1(x) * f_2(x)$ and is defined as*

$$(f_1 * f_2)((x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau) f_2(x-\tau) d\tau. \tag{1.3}$$

2. Algebraic and Topological Structure

Let s_f denote the abscissa of convergence for a Fourier transformable function f as such for all $s > s_f$, $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\&x} dx$ exists in the Lebeque sense and is finite, i.e., $e^{-i\&x} f \epsilon\ l_1(-\infty, \infty) = l$. obviously, we cannot declare that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\&x} dx$$

will exist and will be finite. In fact, s_f can be achieved by a Dedekind cut and as such the behavior at cannot be ensured. It is easy to see that there is no loss of generality if s_f is restricted in $(-\infty, \infty)$ since a function is already in a l_1 -space if its abscissa of convergence is less than zero, for our convenience the set denoted L_τ by is taken to represent the set of Fourier transformable functions throughout this paper

Theorem 2.1. *The L_τ set under the convolution operation is a commutative semi-group.*

Proof. Let L_τ be the set that contains all the Fourier transformable functions. Then commutativity holds in L_τ under the convolution operation * i.e.,

$$(f_1 * f_2)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau) f_2(x-\tau) d\tau$$

Let $u = x - \tau$. So

$$du = -d\tau$$

and

$$\begin{aligned} (f_1 * f_2)(x) &= \frac{1}{\sqrt{2\pi}} \int_{\infty}^{-\infty} f_1(x-u) f_2(u)(-du) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_2(u) f_1(x-u) du \\ &= (f_2 * f_1)(x). \end{aligned}$$

Associativity holds in L_τ under the convolution operation, i.e.,

$$\begin{aligned}(f_1 * f_2) * f_3)(x) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(f_1 * f_2)(\tau)f_3(x-\tau)d\tau \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_1(u)f_2(\tau-u)du)f_3(x-\tau)d\tau \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(u)f_2(\tau-u)f_3(x-\tau)dud\tau \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(u)f_2(\tau-u)f_3(x-\tau)d\tau du \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(u)(\int_{-\infty}^{\infty} f_2(\tau)f_3(x-u-\tau)d\tau)du \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(u)(f_2 * f_3)(x-u)du \\ &= \Big(f_1 * (f_2 * f_3)\Big)(x)\end{aligned}$$

Distributively holds in L_τ under the convolution operation * i.e.,

$$\begin{aligned}\Big(f_1 * (f_2+f_3)\Big)(x) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_1(\tau)(f_2+f_3)(x-\tau)d\tau \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_1(\tau)f_2(x-\tau)d\tau + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_1(\tau)f_3(x-\tau)d\tau \\ &= (f_1 * f_2)(x) + (f_1 * f_3)(x).\end{aligned}$$

Identity property also holds in L_τ under the convolution operation * i.e.,

$$(f * \delta)(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_1(\tau)\delta(x-\tau)d\tau = (\delta * f)(x) = f(x), \tag{2.1}$$

where δ is Kronecker delta. □

Remark 2.2. *Some distributions have an inverse element $K^{(-1)}$ for the convolution, which is defined by*

$$K^{(-1)} * K = \delta \tag{2.2}$$

Corollary 2.3. *If the set L_τ has invertible distributions. Then L_τ form an abelian group under the convolution operation.*

Theorem 2.4. *The L_τ set is an Abelian group with respect to the operation of addition.*

Proof. Let f_1 and f_2 be two functions and let us suppose that they belong to L_τ set. i.e., there exist δ_1 and δ_2 such that $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_1(x)e^{-i\&_1(x)}dx$ and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_2(x)e^{-i\&_2(x)}dx$ exist. Evidently, if we take $\&=\max(\&_1,\&_2)$, then $F(\&_1)+F(\&_2) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(f_1(x)+f_2(x))e^{-i\&(x)}dx$ exists. Thus, the set is closed for addition. The associative property is evident. The null element and additive inverse are respectively the ordinary zero and $-f(x)$. Since $f(x)$ is a Fourier transformable, $-f(x)$ is also so. The commutative property is obvious. Hence the theorem. It becomes now very easy to verify that our L_τ set is a linear system. □

Corollary 2.5. *If L_τ consists of only the positive Fourier transformable functions. Then L_τ forms an abelian semi-group with respect to the operation of addition.*

Now, certain symbols are defined which are to be used throughout this paper.F_s is the class of all.
function in the L_τ set such that k_f = K. Then

$$F_K = F_k^1 \cup F_k^2,$$

where

$$\Big\{f : e^{-kx} f \epsilon l_1[0, \infty)\Big\} = F_k^1, \quad \Big\{f : e^{-kx} f \epsilon l_1[0, \infty)\Big\} = F_k^2.$$

If $s = 0$, then

$$F_0 = F_0 - \cup F_{0+},$$

where $F_{0-} = \{f : k < 0\}$. Hence

$$F_0 = F_{0-} \cup F \cup {F_0}^2.$$

Thus the L_τ set $=UF_r = (U_r {F_r}^1)U({F_r}^2) = F^1 U F^2$ (r being real and ≥ 0)
Again, evidently for $k > 0$ and $t > 0$, we have

$$F_t = e^{(t-k)x} F_k.$$

For $k = 0$, we have

$$F_t = e^{tx} F_0+,$$

i.e.,

$$ {F_t}^1 = e^{tx} F^1{}_0+, \quad {F_t}^2 = e^{tx} F^2{}_0+.$$

Let $l^k[0, \infty))$ be defined as a Banach space with norm.

$$||f|| = \int_{-\infty}^{\infty} e^{-kx} |f(x)| dx < \infty. \tag{2.3}$$

The norm introduces the metric

$$d(f, g) = \int_{-\infty}^{\infty} e^{-kx} |f(x) - g(x)| dx < \infty. \tag{2.4}$$

Definition 2.6. Let f and g be two functions belonging to the L_t. Then a metric for f and g is denoted and defined as follows

$$d(f, g) = |k_f - k_g| + \frac{\int_{-\infty}^{\infty} |e^{-k f x} f - e^{-k g x} g| dx}{1 + \int_{-\infty}^{\infty} |e^{-k f x} - e^{-k g x} g| dx} \tag{2.5}$$

Remark 2.7. k_f can be looked upon as a functional on the L_t. It may be seen that in this topology k_f is a continuous functional.

Now, it can easily be shown that the metric defined above satisfies all the required conditions:
(i) If $f = g$, evidently $d(f, g) = 0$.

Conversely, if $d(f, g) = 0$ then

$$|k_f - k_g| + \frac{\int_{-\infty}^{\infty} |e^{-k_f x} f - e^{-k_g x} g| dx}{1 + \int_{-\infty}^{\infty} |e^{-k_f x} - e^{-k_g x} g| dx} = 0.$$

The two portions being separately positive, they must vanish separately, i.e., $|k_f - k_g| = 0$ giving $k_f = k_g$ and $\int_{-\infty}^{\infty} |e^{-k_f x} f - e^{-k_g x} g| dx = 0$, i.e.

$$e^{-k_f x} f = e^{-k_g x} g.$$

But $k_f = k_g$; hence $f = g$. The property of symmetry and transitivity being very obvious it follows that ρ is a metric.

Remark 2.8. $k_{fn} \Rightarrow k_f$ if $d(f_n, g) \Rightarrow 0$ and $\begin{cases} fn \Rightarrow f \\ k_{fn} \Rightarrow k_f \end{cases} \leftrightarrow g_n \Rightarrow f$
where $g_n \epsilon FK, g_n$ being equal to $exp\{(k_{fn} - k_f)x\} f_n(x)$. This show that $f_n \Rightarrow f$ then there is a sequence $g_n \epsilon FK_f$ such that $g_n \Rightarrow f$ so that any convergent sequence can always be taken to be confined in a given class f_k.

Remark 2.9. $k_f$being a continuous linear functional on the L_τ -space and k_f being equal to $\{f : k_f = k\}$ it follows that every k_f is closed.

Remark 2.10 . In f_k^1 the metric becomes

$$d(f, g) = \int_{-\infty}^{\infty} e^{-kx} |f(x) - g(x)| dx.$$

So that the relative topology in f_k^1 induced by the L_τ -space is the same as the one induced by $l_k[-\infty, \infty)$. Suppose now that f_k is given with its topology as induced by $l_k[-\infty, \infty)$. Then one way of met rising $\cup l_k[0, \infty)$ so that each subspace has the same relative topology as above, is given by our metric.

Remark 2.11. The usual uniform convergence in L_τ -space does not imply convergence as induced by the above metric.
Let us study F_0 alone, but these considerations can easily be extended to F_k .
In F_0 a relation between f and g is defined as:

$$fRg \quad \text{iff} \quad \int_{-\infty}^{\infty} |f(x) - g(x)| dx < \infty. \tag{2.6}$$

Evidently R is an equivalence relation. Obviously since $g(x) = f(x) + [g(x) - f(x)]$, it follows.
that

$$|g(x)| \leq |f(x) - g(x)| + |f(x)|,$$

where $[g(x) - f(x)] \epsilon l_1$. It follows that F_0 is partitioned into disjoint classes and each class is of the form $(f + l_1)$, where

$$\int_{-\infty}^{\infty} |f(x)| dx = \infty.$$

The distance between any two elements of two classes is 1. In fact, these are the elements of the factor space of F_0 with respect to l_1 in F_0 . Thus, the factor space in its quotient topology is discrete. This is not very unnatural since our metric has not made any use of the crucial property of a function $f \epsilon F_0^2$, i.e. ,

$$\int_{-\infty}^{\infty} e^{-kx} |f(x)| dx < \infty, \quad \text{for every } \epsilon > 0. \tag{2.7}$$

It appears that our metric is not sensitive enough for studying the L_τ set. Perhaps a different metric in this way can be considered:

$$d(f, g) = |k_f - k| + \sum_{n=1}^{a} \frac{1}{2^n} \frac{\{\int_0^n |f - g|^p dx\}^{\frac{1}{p}}}{1 + \{\int_0^n |f - g|^p dx\}^{\frac{1}{p}}} \tag{2.8}$$

The metric introduces the L_τ -convergence on each compact subset of reals. Actually, by this metric the distance between two classes does not become unity and the factor space topology will not be discrete.

Theorem 2.12. The L_τ- space is complete with our metric.

Proof. Let f_n be a Cauchy sequence in the L_τ -space. Let $P_1 \subset P$, P being the set of positive integers, be defined as

$$P_1 = \Big\{ n : g_n(x) \epsilon F_0^1 \Big\}, \qquad P_2 \subseteq P = \Big\{ n : g_n(x) \epsilon F_0^2 \Big\}. \tag{2.9}$$

Then neither N_1 nor N_2 can be infinite for that would contradict the fact that $\{f_n\}$ is Cauchy sequence.

$$d(f_n, f_{n+m}) = |K_{f_n} - K_{f_{n+m}}| + \frac{\int_{-\infty}^{\infty} |exp\{-K_{f_n} x\} f_n - exp\{-K_{f_{n+m}} x\} f_{n+m}| dx}{1 + \int_{-\infty}^{\infty} |exp\{-K_{f_n} x\} f_n - kx\} f_{n+m}| dx} \tag{2.10}$$

But since the real no space is complete and $F_0 \_ U F_0^1$ is complete, then $g_n \Rightarrow g$, i.e., $f_n(x) \Rightarrow \{e^{kx} g(x)\}$. If $g_n, n \geq P_0$, belongs to P_0^2 then $g_n - (f)$ for $n \geq P_0$ where $f \epsilon P_0^2$. Then $g_n \Rightarrow f + l_1$, i.e.,

$$\int_0^a |(g_n - f) - (g_{n+m} - f)| dx = \int_0^a |(g_n - g_{n+m})| dx \Rightarrow 0. \tag{2.11}$$

l_1 being complete, $g_n^{'} \Rightarrow g^{'}, g^{'} \epsilon l_1$ and $g_n \Rightarrow f + g^{'}$, i.e.,

$$f_n \Rightarrow e^{sx}(f + g^{'}) \epsilon L_\tau - space.$$

Hence the L_τ - space is complete.

Theorem 2.13. The L_τ space is disconnected.
Proof. It is clear that the L_τ - space= $F^1 U F^2$ and it has just shown that F^1 is complete, and so obviously it is closed. Similarly, F^2 is closed. Hence the L_τ - space is disconnected.

Remark 2.14. Every F_k thus becomes disconnected in its relative topology and $F_k = F_k^1 U F_k^2$, where F_k^1 and F_k^2 are relatively closed in F_k .

Theorem 2.15. (Separability). F^1 is separable in the relative topology.
Proof. In fact, let K_n be a countable dense subset of $[0, \infty)$. Then let $\{f_n\}$ be a dense subset of $\{F_{0-} U F_{0-}^1\}$ which in its relative topology is identical with $l_1[0, \infty)$. Then $\{e^{k_n x} f_m(x)\}$ is a dense subset in F^1 as can easily be seen. Hence we get the proof. □

Now, only the positive functions will be considered. The continuity of the additive operation can be proved in the following manner:
Let k_f, k_g, k, k_{gn} be the abscissas of convergence for f, g, f_n and g_n respectively.

Case I: Let $k_g > k_f$. Then there is a neighborhood of k_g in which there is no element of the sequence k_{fn}, and since k_g is the limit of the sequence k_{gn}. This neighborhood contains all k_{gn} for $n \geq n_0$. Hence for all $n \geq n_0$, the abscissa of convergence of $f_n + g_n$ is k_{gn}:

$$d(f_n + g_n, f + g) = |k_{gn} + k_g| + \int_{-\infty}^{\infty} \Big| exp\{-k_{gn}x\}(f_n + g_n) - exp\{-k_{gn}x\}(f + g) \Big| dx$$

$$< |k_{gn} + k_g| + \int_{-\infty}^{\infty} \Big| exp\{-k_{gn}x\} g_n - exp\{-k_g x\} g \Big| dx + \int_{-\infty}^{\infty} \Big| exp\{-k_{gn}x\} f_n - exp\{-k_g x\} f \Big| dx$$

$$< d(g_n, g) + \int_{-\infty}^{\infty} \Big| exp\{-k_{gn}x + k_{fn}x\} exp\{-k_{fn}x\} f_n$$

$$-exp\{-k_{gn}x + k_{fn}x\} exp\{-k_f x\} f \Big| dx + \int_{-\infty}^{\infty} \Big| exp\{-k_{gn}x + k_{fn}x\} exp\{-k_f x\} f$$

$$-exp\{-k_g x + k_f x\} exp\{-k_f x\} f \Big| dx$$

$$= d(g_n, g) + \int_{-\infty}^{\infty} \Big|exp\{-k_{gn}x + k_{fn}x\}\Big| \times \Big|exp\{-k_{fn}x\}f_n - exp\{-k_f x\}f\Big|dx$$
$$+ \int_{-\infty}^{\infty} \Big|exp\{-k_f x\}f\Big| \ \Big|exp\{-k_{gn}x + k_{fn}x\} - exp\{-k_g x + k_f x\}\Big|dx$$
$$< \rho(g_n, g) + \rho(f_n, f) + \int_{-\infty}^{\infty} \Big|exp\{-k_f x\}f\Big|\Big|exp\{-k_{gn}x + k_{fn}x\}$$
$$-exp\{-k_g x + k_f x\}\Big|dx \Rightarrow 0, \quad as\ n \Rightarrow a,$$

i.e.,

$$f_n + g_n \Rightarrow f + g \quad as \quad n \Rightarrow a.$$

Case II $.k_f = k_g$ In this case in every neighborhood of k_f and similarly in every neighborhood of k_g there are elements of k_{fn} also. But in a certain case $k_{fn+gn} = k_{fn}$, and in other cases k_{gn}. But in all cases, it has seen that

$$d(f_n+g_n, f+g) = |k_{fn}-k_f|+\int_{-\infty}^{\infty} |exp\{-k_{fn}x\}(f_n+g_n)-exp\{-k_f x\}(f+g)dx$$

or

$$|k_{gn}+k_g|+\int_{-\infty}^{\infty} |exp\{-k_{gn}x\}(f_n+g_n)-exp\{-k_g x\}(f+g)|dx < d(g_n, g)+d(f_n, f)$$

$$+\left\{ \begin{array}{l} \int_{-\infty}^{\infty} |exp\{-k_g x\}g||exp\{k_{fn}x + k_{gn}x\} - exp\{-k_f x + k_g x\}|dx \\ \int_{-\infty}^{\infty} |exp\{-k_f x\}f||exp\{-kg_n x + kf_n x\} - exp\{-k_g x + k_f x\}|dx \end{array} \right\} \Rightarrow 0$$

as $n \Rightarrow \infty$, i.e., $(f_n + g_n) \Rightarrow (f + g)$ as $n \Rightarrow \infty$. This shows that

$$(f_n + g_n) \Rightarrow (f + g), \quad f_n \Rightarrow f, \qquad g_n \Rightarrow g.$$

Now the L_τ space can be shown not to be a linear metric space with the metric as introduced above. The property that $a_n f \Rightarrow af$ is not valid for all functions in the space. In fact, if only the set of all positive functions is considered, then that set will form a topological semi-group.

3. **Concluding Remarks**

It is observed that algebraic and topological structures with special properties play a central role in the investigation of the Fourier transformable functions. There is no doubt that the research along this line can be kept up, and indeed, some results in this paper have already made up a foundation for farther exploration concerning the farther progression of the algebraic and topological structures of Fourier transformable functions and their applications in other disciplines of mathematics. The forthcoming study of the algebraic and topological structures of Fourier transformable functions may be the following topics are worth to be taken into account.

(i) To describe the algebra and topology of other integral transforms like Sumudu transform, Fourier transform by using this concept.

(ii) To refer this concept to some other algebraic and topological structures.

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