


## Some algebraic and topological structures of Fourier transformable functions

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**Abstract.** In this work, the set of all functions that are Fourier transformable with regard to their structure both algebraic and topological is taken into account. Certain topological properties of the set of Fourier transformable functions with the help of a metric are described. Also determines the proofs of the statements that the set of all Fourier transformable functions is a commutative semigroup with respect to the convolution operation as well as Abelian group with respect to the operation of addition. Metric for two functions belonging to the set of all Fourier transformable functions is defined and the proof that the Fourier transformable functions space is complete with our metric is given. The separability theorem and that the Fourier transformable functions space is disconnected are also discussed.

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## 1. Introduction

The Fourier transform was first initiated in 1807 and is named after Jean Baptiste Joseph Fourier. Fourier transforms are most well-known and well-established techniques in the areas of engineering and mathematics. Fourier transform technique characterizes the variable as a sum of complex exponentials. Fourier analysis has been utilized in digital image processing and signal processing for the anatomization of a single image as a 2D wave form, plus several other kind of form in particular Image Processing, Signal processing and Quantum mechanics. Fourier analysis also signifies filters, representation, encoding and Transformation, Data Processing and various other fields. Now a day the usage of Fourier transform in different applications has magnificated. Among the various transformation methods used in mathematics, this is one of the simplest due to which this method consumes less time. This transform is extensively used in mechanical system, power distribution system, wireless networks and industries. Generally in power distribution system, this transform is a quick, accurate, and noise-resistant method for modifying power quality disturbances. Furthermore it has vast area of its applications in cell phone and the use of it in the medical sciences. In cell phone, this transform utilizes the signal processing forms and the making of the mobile phone. The Fourier transform is an important tool for solving linear constant coefficient, ordinary or partial differential equations under appropriate initial and boundary value problems in the modern world. It is mainly a linear operator that transforms a function  $f(x)$  into a function  $F(\&)$  with a complex argument ' $\&$ '. The Fourier transform is a transformation from the time domain to the frequency domain used in the study of linear time-invariant systems (electrical circuits, harmonic oscillators, mechanical systems and optical devices). It is mostly related to Laplace transform. However the difference literally lies in that the Laplace transform patch up a function into its moments whereas Fourier transform expresses a function or signal as a series of modes of vibration. For further details (see the references ([1] -[13]).

**Definition 1.1.** (*Fourier Transform*)

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-i\&x} dx, (\& > 0) \quad (1.1)$$

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**Definition 1.2.** (*Inverse Fourier Transform*)

$$F^{-1}\{f(\&x)\} = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\&x)e^{i\&x} d\&x, (\&x > 0) \quad (1.2)$$

**Definition 1.3.** (*Convolution of Fourier Transform*) *The convolution of  $f_1(x)$  and  $f_2(x)$  where  $f_1(x)$  and  $f_2(x)$  are the Fourier transformable functions is represented by  $f_1(x) * f_2(x)$  and is defined as*

$$(f_1 * f_2)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau)f_2(x - \tau)d\tau. \quad (1.3)$$

## 2. Algebraic and Topological Structure

Let  $s_f$  denote the abscissa of convergence for a Fourier transformable function  $f$  as such for all  $s > s_f$ ,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-i\&x} dx$  exists in the Lebeque sense and is finite, i.e.,  $e^{-i\&x} f \in l_1(-\infty, \infty) = l$ . obviously, we cannot declare that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-i\&x} dx$$

will exist and will be finite. In fact,  $s_f$  can be achieved by a Dedekind cut and as such the behavior at cannot be ensured. It is easy to see that there is no loss of generality if  $s_f$  is restricted in  $(-\infty, \infty)$  since a function is already in a  $l_1$  -space if its abscissa of convergence is less than zero, for our convenience the set denoted  $L_\tau$  by is taken to represent the set of Fourier transformable functions throughout this paper

**Theorem 2.1.** *The  $L_\tau$  set under the convolution operation is a commutative semi-group.*

*Proof.* Let  $L_\tau$  be the set that contains all the Fourier transformable functions. Then commutativity holds in  $L_\tau$  under the convolution operation ‘\*’ i.e.,

$$(f_1 * f_2)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau)f_2(x - \tau)d\tau$$

Let  $u = x - \tau$ . So

$$du = -d\tau$$

and

$$\begin{aligned} (f_1 * f_2)(x) &= \frac{1}{\sqrt{2\pi}} \int_{\infty}^{-\infty} f_1(x - u)f_2(u)(-du) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_2(u)f_1(x - u)du \\ &= (f_2 * f_1)(x). \end{aligned}$$

Associativity holds in  $L_\tau$  under the convolution operation, i.e.,

$$\begin{aligned}
(f_1 * f_2) * f_3(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (f_1 * f_2)(\tau) f_3(x - \tau) d\tau \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(u) f_2(\tau - u) du \right) f_3(x - \tau) d\tau \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(u) f_2(\tau - u) f_3(x - \tau) du d\tau \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(u) f_2(\tau - u) f_3(x - \tau) d\tau du \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(u) \left( \int_{-\infty}^{\infty} f_2(\tau) f_3(x - u - \tau) d\tau \right) du \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(u) (f_2 * f_3)(x - u) du \\
&= (f_1 * (f_2 * f_3))(x)
\end{aligned}$$

Distributively holds in  $L_\tau$  under the convolution operation “\*” i.e.,

$$\begin{aligned}
(f_1 * (f_2 + f_3))(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau) (f_2 + f_3)(x - \tau) d\tau \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau) f_2(x - \tau) d\tau + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau) f_3(x - \tau) d\tau \\
&= (f_1 * f_2)(x) + (f_1 * f_3)(x).
\end{aligned}$$

Identity property also holds in  $L_\tau$  under the convolution operation “\*” i.e.,

$$(f * \delta)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(\tau) \delta(x - \tau) d\tau = (\delta * f)(x) = f(x), \quad (2.1)$$

where  $\delta$  is Kronecker delta.  $\square$

**Remark 2.2.** Some distributions have an inverse element  $K^{(-1)}$  for the convolution, which is defined by

$$K^{(-1)} * K = \delta \quad (2.2)$$

**Corollary 2.3.** If the set  $L_\tau$  has invertible distributions. Then  $L_\tau$  form an abelian group under the convolution operation.

**Theorem 2.4.** The  $L_\tau$  set is an Abelian group with respect to the operation of addition.

*Proof.* Let  $f_1$  and  $f_2$  be two functions and let us suppose that they belong to  $L_\tau$  set. i.e., there exist  $\delta_1$  and  $\delta_2$  such that  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_1(x) e^{-i\&_1(x)} dx$  and  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_2(x) e^{-i\&_2(x)} dx$  exist. Evidently, if we take  $\& = \max(\&_1, \&_2)$ , then  $F(\&_1) + F(\&_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (f_1(x) + f_2(x)) e^{-i\&(x)} dx$  exists. Thus, the set is closed for addition. The associative property is evident. The null element and

additive inverse are respectively the ordinary zero and  $-f(x)$ . Since  $f(x)$  is a Fourier transformable,  $-f(x)$  is also so. The commutative property is obvious. Hence the theorem. It becomes now very easy to verify that our  $L_\tau$  set is a linear system.  $\square$

**Corollary 2.5.** *If  $L_\tau$  consists of only the positive Fourier transformable functions. Then  $L_\tau$  forms an abelian semi-group with respect to the operation of addition.*

Now, certain symbols are defined which are to be used throughout this paper.  $F_s$  is the class of all

function in the  $L_\tau$  set such that  $k_f = K$ . Then

$$F_K = F_k^1 \cup F_k^2,$$

where

$$\left\{ f : e^{-kx} f \in L_1[0, \infty) \right\} = F_k^1, \quad \left\{ f : e^{-kx} f \in L_1[0, \infty) \right\} = F_k^2.$$

If  $s = 0$ , then

$$F_0 = F_{0-} \cup F_{0+},$$

where  $F_{0-} = \{f : k < 0\}$ . Hence

$$F_0 = F_{0-} \cup F \cup F_{0+}^2.$$

Thus the  $L_\tau$  set  $= UF_r = (U_r F_r^{-1})U(F_r^2) = F^1 U F^2$  ( $r$  being real and  $\geq 0$ )  
Again, evidently for  $k > 0$  and  $t > 0$ , we have

$$F_t = e^{(t-k)x} F_k.$$

For  $k = 0$ , we have

$$F_t = e^{tx} F_{0+},$$

i.e.,

$$F_t^1 = e^{tx} F_{0+}^1, \quad F_t^2 = e^{tx} F_{0+}^2.$$

Let  $l^k[0, \infty)$  be defined as a Banach space with norm.

$$\|f\| = \int_{-\infty}^{\infty} e^{-kx} |f(x)| dx < \infty. \quad (2.3)$$

The norm introduces the metric

$$d(f, g) = \int_{-\infty}^{\infty} e^{-kx} |f(x) - g(x)| dx < \infty. \quad (2.4)$$

**Definition 2.6.** Let  $f$  and  $g$  be two functions belonging to the  $L_t$ . Then a metric for  $f$  and  $g$  is denoted and defined as follows

$$d(f, g) = |k_f - k_g| + \frac{\int_{-\infty}^{\infty} |e^{-kfx} f - e^{-kgx} g| dx}{1 + \int_{-\infty}^{\infty} |e^{-kfx} f - e^{-kgx} g| dx} \quad (2.5)$$

**Remark 2.7.**  $k_f$  can be looked upon as a functional on the  $L_t$ . It may be seen that in this topology  $k_f$  is a continuous functional.

Now, it can easily be shown that the metric defined above satisfies all the required conditions:

(i) If  $f = g$ , evidently  $d(f, g) = 0$ .

Conversely, if  $d(f, g) = 0$  then

$$|k_f - k_g| + \frac{\int_{-\infty}^{\infty} |e^{-kfx}f - e^{-kgx}g|dx}{1 + \int_{-\infty}^{\infty} |e^{-kfx}f - e^{-kgx}g|dx} = 0.$$

The two portions being separately positive, they must vanish separately, i.e.,  $|k_f - k_g| = 0$  giving  $k_f = k_g$  and  $\int_{-\infty}^{\infty} |e^{-kfx}f - e^{-kgx}g|dx = 0$ , i.e.

$$e^{-kfx}f = e^{-kgx}g.$$

But  $k_f = k_g$ ; hence  $f = g$ . The property of symmetry and transitivity being very obvious it follows that  $\rho$  is a metric.

**Remark 2.8.**  $k_{f_n} \Rightarrow k_f$  if  $d(f_n, g) \Rightarrow 0$  and  $\begin{cases} f_n \Rightarrow f \\ k_{f_n} \Rightarrow k_f \end{cases} \Leftrightarrow g_n \Rightarrow f$

where  $g_n \in FK$ ,  $g_n$  being equal to  $\exp\{(k_{f_n} - k_f)x\}f_n(x)$ . This shows that  $f_n \Rightarrow f$  then there is a sequence  $g_n \in FK_f$  such that  $g_n \Rightarrow f$  so that any convergent sequence can always be taken to be confined in a given class  $f_k$ .

**Remark 2.9.**  $k_f$  being a continuous linear functional on the  $L_\tau$ -space and  $k_f$  being equal to  $\{f : k_f = k\}$  it follows that every  $k_f$  is closed.

**Remark 2.10 .** In  $f_k^1$  the metric becomes

$$d(f, g) = \int_{-\infty}^{\infty} e^{-kx}|f(x) - g(x)|dx.$$

So that the relative topology in  $f_k^1$  induced by the  $L_\tau$ -space is the same as the one induced by  $l_k[-\infty, \infty)$ . Suppose now that  $f_k$  is given with its topology as induced by  $l_k[-\infty, \infty)$ . Then one way of met rising  $\cup l_k[0, \infty)$  so that each subspace has the same relative topology as above, is given by our metric.

**Remark 2.11.** The usual uniform convergence in  $L_\tau$ -space does not imply convergence as induced by the above metric.

Let us study  $F_0$  alone, but these considerations can easily be extended to  $F_k$ . In  $F_0$  a relation between  $f$  and  $g$  is defined as:

$$fRg \text{ iff } \int_{-\infty}^{\infty} |f(x) - g(x)|dx < \infty. \quad (2.6)$$

Evidently  $R$  is an equivalence relation. Obviously since  $g(x) = f(x) + [g(x) - f(x)]$ , it follows.

that

$$|g(x)| \leq |f(x) - g(x)| + |f(x)|,$$

where  $[g(x) - f(x)] \in l_1$ . It follows that  $F_0$  is partitioned into disjoint classes and each class is of the form  $(f + l_1)$ , where

$$\int_{-\infty}^{\infty} |f(x)| dx = \infty.$$

The distance between any two elements of two classes is 1. In fact, these are the elements of the factor space of  $F_0$  with respect to  $l_1$  in  $F_0$ . Thus, the factor space in its quotient topology is discrete. This is not very unnatural since our metric has not made any use of the crucial property of a function  $f \in F_0^2$ , i.e.,

$$\int_{-\infty}^{\infty} e^{-kx} |f(x)| dx < \infty, \quad \text{for every } \epsilon > 0. \quad (2.7)$$

It appears that our metric is not sensitive enough for studying the  $L_\tau$  set. Perhaps a different metric in this way can be considered:

$$d(f, g) = |k_f - k| + \sum_{n=1}^a \frac{1}{2^n} \frac{\{\int_0^n |f - g|^p dx\}^{\frac{1}{p}}}{1 + \{\int_0^n |f - g|^p dx\}^{\frac{1}{p}}} \quad (2.8)$$

The metric introduces the  $L_\tau$ -convergence on each compact subset of reals. Actually, by this metric the distance between two classes does not become unity and the factor space topology will not be discrete.

**Theorem 2.12.** The  $L_\tau$ -space is complete with our metric.

*Proof.* Let  $f_n$  be a Cauchy sequence in the  $L_\tau$ -space. Let  $P_1 \subset P$ ,  $P$  being the set of positive integers, be defined as

$$P_1 = \{n : g_n(x) \in F_0^1\}, \quad P_2 \subseteq P = \{n : g_n(x) \in F_0^2\}. \quad (2.9)$$

Then neither  $N_1$  nor  $N_2$  can be infinite for that would contradict the fact that  $\{f_n\}$  is Cauchy sequence.

$$d(f_n, f_{n+m}) = |K_{f_n} - K_{f_{n+m}}| + \frac{\int_{-\infty}^{\infty} |\exp\{-K_{f_n}x\}f_n - \exp\{-K_{f_{n+m}}x\}f_{n+m}| dx}{1 + \int_{-\infty}^{\infty} |\exp\{-K_{f_n}x\}f_n - \exp\{-K_{f_{n+m}}x\}f_{n+m}| dx} \quad (2.10)$$

But since the real no space is complete and  $F_0 - UF_0^1$  is complete, then  $g_n \Rightarrow g$ , i.e.,  $f_n(x) \Rightarrow \{e^{kx}g(x)\}$ . If  $g_n, n \geq P_0$ , belongs to  $P_0^2$  then  $g_n - (f)$  for  $n \geq P_0$  where  $f \in P_0^2$ . Then  $g_n \Rightarrow f + l_1$ , i.e.,

$$\int_0^a |(g_n - f) - (g_{n+m} - f)| dx = \int_0^a |(g_n - g_{n+m})| dx \Rightarrow 0. \quad (2.11)$$

$l_1$  being complete,  $g'_n \Rightarrow g', g' \in l_1$  and  $g_n \Rightarrow f + g'$ , i.e.,

$$f_n \Rightarrow e^{sx}(f + g') \in L_\tau - \text{space}.$$

Hence the  $L_\tau$  - space is complete.

**Theorem 2.13.** The  $L_\tau$  space is disconnected.

*Proof.* It is clear that the  $L_\tau$  - space =  $F^1 U F^2$  and it has just shown that  $F^1$  is complete, and so obviously it is closed. Similarly,  $F^2$  is closed. Hence the  $L_\tau$  - space is disconnected.

**Remark 2.14.** Every  $F_k$  thus becomes disconnected in its relative topology and  $F_k = F_k^1 U F_k^2$ , where  $F_k^1$  and  $F_k^2$  are relatively closed in  $F_k$ .

**Theorem 2.15.** (Separability).  $F^1$  is separable in the relative topology.

*Proof.* In fact, let  $K_n$  be a countable dense subset of  $[0, \infty)$ . Then let  $\{f_n\}$  be a dense subset of  $\{F_{0-} U F_{0-}^1\}$  which in its relative topology is identical with  $l_1[0, \infty)$ . Then  $\{e^{k_n x} f_m(x)\}$  is a dense subset in  $F^1$  as can easily be seen. Hence we get the proof.  $\square$

Now, only the positive functions will be considered. The continuity of the additive operation can be proved in the following manner:

Let  $k_f, k_g, k, k_{g_n}$  be the abscissas of convergence for  $f, g, f_n$  and  $g_n$  respectively.

Case I: Let  $k_g > k_f$ . Then there is a neighborhood of  $k_g$  in which there is no element of the sequence  $k_{f_n}$ , and since  $k_g$  is the limit of the sequence  $k_{g_n}$ . This neighborhood contains all  $k_{g_n}$  for  $n \geq n_0$ . Hence for all  $n \geq n_0$ , the abscissa of convergence of  $f_n + g_n$  is  $k_{g_n}$ :

$$\begin{aligned} d(f_n + g_n, f + g) &= |k_{g_n} + k_g| + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x\}(f_n + g_n) - \exp\{-k_{g_n}x\}(f + g) \right| dx \\ &< |k_{g_n} + k_g| + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x\}g_n - \exp\{-k_gx\}g \right| dx + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x\}f_n - \exp\{-k_gx\}f \right| dx \\ &< d(g_n, g) + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x + k_{f_n}x\} \exp\{-k_{f_n}x\}f_n \right. \\ &\quad \left. - \exp\{-k_{g_n}x + k_{f_n}x\} \exp\{-k_{f_n}x\}f \right| dx + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x + k_{f_n}x\} \exp\{-k_{f_n}x\}f \right. \\ &\quad \left. - \exp\{-k_gx + k_{f_n}x\} \exp\{-k_{f_n}x\}f \right| dx \end{aligned}$$



$$\begin{aligned}
&= d(g_n, g) + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x + k_{f_n}x\} \right| \times \left| \exp\{-k_{f_n}x\}f_n - \exp\{-k_f x\}f \right| dx \\
&\quad + \int_{-\infty}^{\infty} \left| \exp\{-k_f x\}f \right| \left| \exp\{-k_{g_n}x + k_{f_n}x\} - \exp\{-k_g x + k_f x\} \right| dx \\
&\quad < \rho(g_n, g) + \rho(f_n, f) + \int_{-\infty}^{\infty} \left| \exp\{-k_f x\}f \right| \left| \exp\{-k_{g_n}x + k_{f_n}x\} \right. \\
&\quad \quad \left. - \exp\{-k_g x + k_f x\} \right| dx \Rightarrow 0, \quad \text{as } n \Rightarrow a,
\end{aligned}$$

i.e.,

$$f_n + g_n \Rightarrow f + g \quad \text{as } n \Rightarrow a.$$

Case II  $k_f = k_g$  In this case in every neighborhood of  $k_f$  and similarly in every neighborhood of  $k_g$  there are elements of  $k_{f_n}$  also. But in a certain case  $k_{f_n+g_n} = k_{f_n}$ , and in other cases  $k_{g_n}$ . But in all cases, it has been seen that

$$d(f_n + g_n, f + g) = |k_{f_n} - k_f| + \int_{-\infty}^{\infty} \left| \exp\{-k_{f_n}x\}(f_n + g_n) - \exp\{-k_f x\}(f + g) \right| dx$$

or

$$\begin{aligned}
&|k_{g_n} + k_g| + \int_{-\infty}^{\infty} \left| \exp\{-k_{g_n}x\}(f_n + g_n) - \exp\{-k_g x\}(f + g) \right| dx < d(g_n, g) + d(f_n, f) \\
&+ \left\{ \begin{aligned} &\int_{-\infty}^{\infty} \left| \exp\{-k_g x\}g \right| \left| \exp\{k_{f_n}x + k_{g_n}x\} - \exp\{-k_f x + k_g x\} \right| dx \\ &\int_{-\infty}^{\infty} \left| \exp\{-k_f x\}f \right| \left| \exp\{-k_{g_n}x + k_{f_n}x\} - \exp\{-k_g x + k_f x\} \right| dx \end{aligned} \right\} \Rightarrow 0
\end{aligned}$$

as  $n \Rightarrow \infty$ , i.e.,  $(f_n + g_n) \Rightarrow (f + g)$  as  $n \Rightarrow \infty$ . This shows that

$$(f_n + g_n) \Rightarrow (f + g), \quad f_n \Rightarrow f, \quad g_n \Rightarrow g.$$

Now the  $L_\tau$  space can be shown not to be a linear metric space with the metric as introduced above. The property that  $a_n f \Rightarrow a f$  is not valid for all functions in the space. In fact, if only the set of all positive functions is considered, then that set will form a topological semi-group.

### 3. Concluding Remarks

It is observed that algebraic and topological structures with special properties play a central role in the investigation of the Fourier transformable functions. There is no doubt that the research along this line can be kept up, and indeed, some results in this paper have already made up a foundation for farther exploration concerning the farther progression of the algebraic and topological structures of Fourier transformable functions and their applications in other disciplines of mathematics. The forthcoming study of the algebraic and topological structures of Fourier transformable functions may be the following topics are worth to be taken into account.

(i) To describe the algebra and topology of other integral transforms like Sumudu transform, Fourier transform by using this concept.

(ii) To refer this concept to some other algebraic and topological structures.

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