

## On six dimensional Finsler spaces

P. K. Dwivedi<sup>a</sup>, C. K. Mishra<sup>a</sup> and Sachin Kumar<sup>a\*</sup> 

<sup>a</sup>Department of Mathematics and Statistics, Dr. Rammanohar Lohia  
Avadh University, Ayodhya (U.P.), India

E-mail: [drpkdwivedi@yahoo.co.in](mailto:drpkdwivedi@yahoo.co.in)

E-mail: [chayankumarmishra@gmail.com](mailto:chayankumarmishra@gmail.com)

E-mail: [skumar17011997@gmail.com](mailto:skumar17011997@gmail.com)

**Abstract.** The objective of this research paper is to comprehensively explore the main scalars within the context of the six dimensional Finsler space. This investigation leverages both  $h$ - connection vectors and  $v$ - connection vectors. Additionally we have introduced the  $T$ - condition and  $v$ - curvature tensor  $S_{hijk}$  and express them in an extended form in relation to scalars and tensors in terms of main scalars.

**Keywords:** Six- dimensional Finsler space,  $h$ - connection vectors,  $v$ - connection vectors,  $T$ - condition.

### 1. Introduction

A theory of  $n$ - dimensional Finsler space studied by Matsumoto and Miron [1, 2, 3] and is called 'Miron Frame'. The Miron frame for six dimensional Finsler space is constructed by the unit vectors  $(e_1^i, e_2^i, e_3^i, e_4^i, e_5^i, e_6^i)$ . The first vector  $e_1^i$  is the normalized supporting element  $l^i$  and second vector  $e_2^i$  is the normalized torsion vector  $m^i = \frac{C^i}{C}$  where  $C$  is the length of torsion vector

---

\* Corresponding author

AMS 2020 Mathematics Subject Classification: 53B40, 53C30

This work is licensed under a [Creative Commons Attribution-NonCommercial 4.0 International License](https://creativecommons.org/licenses/by-nc/4.0/).

Copyright © 2024 The Author(s). Published by University of Mohaghegh Ardabili

$C^i$ . The third vector  $e_{(3)}^i = n^i$ , fourth vector  $e_{(4)}^i = p^i$ , fifth vector  $e_{(5)}^i = q^i$  and the sixth vector  $e_{(6)}^i = r^i$  are constructed by the Matsumoto and Miron [2]. Many investigators such as M. K. Gupta, P. N. Pandey [4] and S. K. Tiwari, Anamika Rai [7] have studied on four and five-dimensional Finsler spaces. P. K. Dwivedi, S. C. Rastogi and A. K. Dwivedi [6] have initially studied Finsler space in terms of scalars.

The purpose of the present investigation is to study the six- dimensional Finsler spaces satisfying  $T$ - conditions and  $V$ - curvature tensors in terms of main scalars with  $h$ - and  $v$ - connection vectors.

## 2. Preliminaries

In view of [2]  $l_i = \dot{\partial}_i L$ ,  $g_{ij} = \frac{1}{2} \dot{\partial}_i \dot{\partial}_j L^2$ ,  $h_{ij} = L \dot{\partial}_i \dot{\partial}_j L$  and  $C_{ijk} = \frac{1}{2} \dot{\partial}_k g_{ij}$  respectively. The  $h$ - and  $v$ - covariant derivatives of a covariant vector  $X_i(x, y)$  with respect to Cartan connection are given by

$$X_{i/j} = \partial_j X_i - (\dot{\partial}_h X_i) G_j^h - F_{ij}^r X_r$$

and

$$X_i/j = \dot{\partial}_j X_i - C_{ij}^r X_r$$

With respect to frame [2], the scalar components of an arbitrary tensor  $T_j^i$  are defined by

$$T_{\alpha\beta} = T_j^i e_{(\alpha)}^i e_{(\beta)}^j \quad (2.1)$$

From this we get

$$T_j^i = T_{\alpha\beta} e_{(\alpha)}^i e_{(\beta)}^j \quad (2.2)$$

where summation convention is also applied to greek indices. The scalar components of the metric tensor  $g_{ij}$  are  $g_{\alpha\beta}$  [4] and Angular metric tensor  $h_{ij}$  [7] of  $F^6$  are given by

$$\begin{cases} g_{ij} = l_i l_j + m_i m_j + n_i n_j + p_i p_j + q_i q_j + r_i r_j, \\ h_{ij} = m_i m_j + n_i n_j + p_i p_j + q_i q_j + r_i r_j \end{cases} \quad (2.3)$$

Let  $H_{\alpha)\beta\gamma}$  and  $V_{\alpha)\beta\gamma}$  be scalar components of the  $h$ - and  $v$ - covariant derivative  $e_{(\alpha)}^i/j$  and  $e_{(\alpha)}^i/j$  respectively of the vector  $e_{(\alpha)}^i$ , then

$$e_{(\alpha)}^i/j = H_{\alpha)\beta\gamma} e_{(\beta)}^i e_{(\gamma)}^j + L e_{(\alpha)}^i = V_{\alpha)\beta\gamma} e_{(\beta)}^i e_{(\gamma)}^j \quad (2.4)$$

The  $h$ - covariant derivative for tensor  $X_i$  is given in above equation.  $H_{\alpha)\beta\gamma}$  and  $V_{\alpha)\beta\gamma}$ , are called  $h$ - and  $v$ -connection scalars respectively and positively homogeneous of degree '0' in  $y$ . Orthogonality of the Miron Frame[2] yields

$$H_{\alpha)\beta\gamma} = -H_{\beta)\alpha\gamma} \text{ and } V_{\alpha)\beta\gamma} = -V_{\beta)\alpha\gamma} \quad (2.5)$$

Also, we have  $H_{1)\beta\gamma} = 0$  and  $V_{1)\beta\gamma} = \delta_{\beta\gamma} - \delta_{1\beta} \delta_{1\gamma}$

Now we define  $h$ -connection and  $v$ - connection vector fields:

$$\begin{aligned}
H_{2)3\beta}e_{\beta}^j &= h_j = -H_{3)2\beta}e_{\beta}^j, H_{3)4\beta}e_{\beta}^j = j_j = -H_{4)3\beta}e_{\beta}^j, \\
H_{4)2\beta}e_{\beta}^j &= k_j = -H_{2)4\beta}e_{\beta}^j, H_{5)2\beta}e_{\beta}^j = p_j = -H_{2)5\beta}e_{\beta}^j, \\
H_{5)3\beta}e_{\beta}^j &= q_j = -H_{3)5\beta}e_{\beta}^j, H_{5)4\beta}e_{\beta}^j = h'_j = -H_{4)5\beta}e_{\beta}^j, \\
H_{6)2\beta}e_{\beta}^j &= j'_j = -H_{2)6\beta}e_{\beta}^j, H_{6)3\beta}e_{\beta}^j = k'_j = -H_{3)6\beta}e_{\beta}^j, \\
H_{6)4\beta}e_{\beta}^j &= p'_j = -H_{4)6\beta}e_{\beta}^j, H_{6)5\beta}e_{\beta}^j = q'_j = -H_{5)6\beta}e_{\beta}^j,
\end{aligned} \tag{2.6}$$

and

$$\begin{aligned}
V_{2)3\beta}e_{\beta}^j &= u_j = -V_{3)2\beta}e_{\beta}^j, V_{3)4\beta}e_{\beta}^j = v_j = -V_{4)3\beta}e_{\beta}^j, \\
V_{4)2\beta}e_{\beta}^j &= w_j = -V_{2)4\beta}e_{\beta}^j, V_{5)2\beta}e_{\beta}^j = r_j = -V_{2)5\beta}e_{\beta}^j, \\
V_{5)3\beta}e_{\beta}^j &= s_j = -V_{3)5\beta}e_{\beta}^j, V_{5)4\beta}e_{\beta}^j = u'_j = -V_{4)5\beta}e_{\beta}^j, \\
V_{6)2\beta}e_{\beta}^j &= v'_j = -V_{2)6\beta}e_{\beta}^j, V_{6)3\beta}e_{\beta}^j = w'_j = -V_{3)6\beta}e_{\beta}^j, \\
V_{6)4\beta}e_{\beta}^j &= r'_j = -V_{4)6\beta}e_{\beta}^j, V_{6)5\beta}e_{\beta}^j = s'_j = -V_{5)6\beta}e_{\beta}^j,
\end{aligned} \tag{2.7}$$

The vector fields  $h_j, j_j, k_j, p_j, q_j, h'_j, j'_j, k'_j, p'_j$  and  $q'_j$  are called  $h$ -connection vectors and the vector fields  $u_j, v_j, w_j, r_j, s_j, u'_j, v'_j, w'_j, r'_j$ , and  $s'_j$  are called  $v$ -connection vectors. The scalars  $H_{2)3\beta}, H_{3)4\beta}, H_{4)2\beta}, H_{5)2\beta}, H_{5)3\beta}, H_{5)4\beta}, H_{6)2\beta}, H_{6)3\beta}, H_{6)4\beta}, H_{6)5\beta}$  and  $V_{2)3\beta}, V_{3)4\beta}, V_{4)2\beta}, V_{5)2\beta}, V_{5)3\beta}, V_{5)4\beta}, V_{6)2\beta}, V_{6)3\beta}, V_{6)4\beta}, V_{6)5\beta}$  are considered as the scalar components  $h_j, j_j, k_j, p_j, q_j, h'_j, j'_j, k'_j, p'_j, q'_j$  and  $u_j, v_j, w_j, r_j, s_j, u'_j, v'_j, w'_j, r'_j, s'_j$  of the  $h$ - and  $v$ -connection vectors respectively with respect to the orthonormal frame. Let  $C_{\alpha\beta\gamma}$  are the scalar components of  $LC_{ijk}$ , then

$$LC_{ijk} = C_{\alpha\beta\gamma}e_{\alpha}ie_{\beta}je_{\gamma}k \tag{2.8}$$

Then, we have from [6]

$$\begin{aligned}
LC_{ijk} = & C_{(222)}m_i m_j m_k + C_{(233)}n_{(1)}|n_{(1)j}n_{(1)k} + C_{(244)}n_{(2)}|n_{(2)j}n_{(2)k} + \\
& C_{(255)}n_{(3)}|n_{(3)j}n_{(3)k} + C_{(266)}n_{(4)}|n_{(4)j}n_{(4)k} \\
& + \sum_{(ijk)} [C_{(322)}m_i m_j n_{(1)k} + C_{(333)}m_i m_j n_{(2)k} + \\
& C_{(344)}m_i m_j n_{(3)k} + C_{(355)}m_i m_j n_{(4)k} + C_{(366)}n_{(1)}|n_{(1)j}m_k + \\
& C_{(422)}n_{(1)}|n_{(1)j}n_{(2)k} + C_{(433)}n_{(1)}|n_{(1)j}n_{(3)k} \\
& + C_{(444)}n_{(1)}|n_{(1)j}n_{(4)k} + C_{(455)}n_{(2)}|n_{(2)j}m_k + \\
& C_{(466)}n_{(2)}|n_{(2)j}n_{(1)k} + C_{(522)}n_{(2)}|n_{(2)j}n_{(3)k} \\
& + C_{(533)}n_{(2)}|n_{(2)j}n_{(4)k} + C_{(544)}n_{(3)}|n_{(3)j}m_k + \\
& C_{(555)}n_{(3)}|n_{(3)j}n_{(1)k} + C_{(566)}n_{(3)}|n_{(3)j}n_{(2)k} + \\
& C_{(622)}n_{(3)}|n_{(3)j}n_{(4)k} + C_{(633)}n_{(4)}|n_{(4)j}m_k + C_{(644)}n_{(4)}|n_{(4)j}n_{(1)k} \\
& + C_{(655)}n_{(4)}|n_{(4)j}n_{(2)k} + C_{(666)}n_{(4)}|n_{(4)j}n_{(3)k} + C_{(234)}m_i(n_{(1)j}n_{(2)k} + n_{(1)k}n_{(4)j}) + \\
& C_{(235)}m_i(n_{(1)j}n_{(3)k} + n_{(1)k}n_{(3)j}) + C_{(236)}m_i(n_{(1)j}n_{(4)k} + n_{(1)k}n_{(4)j}) + \\
& C_{(245)}m_i(n_{(2)j}n_{(3)k} + n_{(2)k}n_{(3)j}) + C_{(246)}m_i(n_{(2)j}n_{(4)k} + n_{(2)k}n_{(4)j}) + \\
& C_{(256)}m_i(n_{(3)j}n_{(4)k} + n_{(3)k}n_{(4)j}) + C_{(345)}n_{(1)}|(n_{(2)j}n_{(3)k} + n_{(2)k}n_{(3)j}) + \\
& C_{(346)}n_{(1)}i(n_{(2)j}n_{(4)k} + n_{(2)k}n_{(4)j}) + C_{(356)}n_{(1)}|(n_{(3)j}n_{(4)k} + n_{(3)k}n_{(4)j}) + \\
& C_{(456)}n_{(2)}i(n_{(3)j}n_{(4)k} + n_{(3)k}n_{(4)j})]
\end{aligned} \tag{2.9}$$

The main scalars of six dimensional Finsler space are given by [6]

$$\begin{aligned}
C_{(222)} = A, C_{(233)} = B, C_{(244)} = C, C_{(255)} = D, C_{(266)} = E, C_{(333)} = F, \\
C_{(344)} = G, C_{(355)} = H, C_{(366)} = I, C_{(433)} = J, C_{(444)} = K, C_{(455)} = L, \\
C_{(466)} = M, C_{(533)} = N, C_{(544)} = A', C_{(555)} = B', C_{(566)} = C', \\
C_{(633)} = D', C_{(644)} = E', C_{(655)} = F', C_{(666)} = G', C_{(234)} = H', \\
C_{(235)} = I', C_{(236)} = J', C_{(245)} = K', C_{(246)} = L', C_{(256)} = M', C_{(345)} = N', \\
C_{(346)} = A'', C_{(356)} = B'', C_{(456)} = C'', \\
\text{We also have, } A + J + N + D', +H' = LC, C_{(223)} = -(B + A' + E' + I'), \\
C_{(224)} = -(C + G + F' + J'), C_{(225)} = -(D + H + L + K'), \\
C_{(226)} = -(E + I + M + C'),
\end{aligned} \tag{2.10}$$

where  $LC$  is called the unified main scalar.

Taking  $h$ -covariant derivative of vector field  $T_j^i$  with respect to  $k$ , we get

$$T_{j/k}^i = (\delta_k T_{\alpha\beta})e_{\alpha}^i e_{\beta}j + T_{\alpha\beta}e_{\alpha}^i /_k e_{\beta}j + T_{\alpha\beta}e_{\alpha}^i e_{\beta}j /_k \tag{2.11}$$

where  $\delta_k = \partial_k G_k^r \partial_r$ . If  $T_{\alpha\beta,\gamma}$  are the scalar components of  $T_{j/k}^i$ , i.e

$$T_{j/k}^i = T_{\alpha\beta,\gamma} e_\alpha^i e_\beta)_j, \quad (2.12)$$

then we obtain

$$T_{\alpha\beta,\gamma} = (\delta_k T_{\alpha\beta}) e_\gamma^k + T_{\mu\beta} H_{\mu)\alpha\gamma} + T_{\alpha\mu} H_{\mu)\beta\gamma} \quad (2.13)$$

Similarly, if  $T_{\alpha\beta;\gamma}$  are the scalar components of  $LT_j^i/k$ , i.e,

$$LT_j^i/k = T_{\alpha\beta;\gamma} e_\alpha^i e_\beta)_j e_\gamma)_k \quad (2.14)$$

then we get

$$T_{\alpha\beta;\gamma} = L(\partial_k T_{\alpha\beta}) e_\gamma^k + T_{\mu\beta} V_{\mu)\alpha\gamma} + T_{\alpha\mu} V_{\mu)\beta\gamma} \quad (2.15)$$

Equation (2.13) and (2.14) are called  $h$ - and  $v$ - scalar derivatives of the component  $T_{\alpha\beta}$  of  $T_j^i$ .

### 3. $T$ -Condition

M. Matsumoto, H. Shimada [5] and M. K. Gupta, P. N. Pandey [4] have been studied  $T$ - tensor defined as

$$T_{hijk} = LC_{ij/k} + C_{hij} l_k + C_{hik} l_j + C_{hkj} l_i + C_{kij} l_h \quad (3.1)$$

It is completely symmetric in its indices. A Finsler space is said to be satisfy  $T$ - condition if the  $T$ -tensor  $T_{hijk}$  vanishes identically.

From (2.8) and (2.14), it follows that

$$L^2 C_{hij/k} + LC_{hij} l_k = C_{\alpha\beta\gamma;\delta} e_\alpha)_h e_\beta)_i e_\gamma)_j e_\delta)_k, \quad (3.2)$$

which implies

$$L^2 C_{hij/k} = (C_{\alpha\beta\gamma;\delta} - C_{\alpha\beta\gamma} \delta/\delta) e_\alpha)_h e_\beta)_i e_\gamma)_j e_\delta)_k \quad (3.3)$$

Therefore the scalar components  $T_{\alpha\beta\gamma\delta}$  of  $LT_{hijk}$  are given by

$$T_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma;\delta} + \delta/\alpha C_{\beta\gamma\delta} + \delta/\beta C_{\alpha\gamma\delta} + \delta/\gamma C_{\alpha\beta\delta} \quad (3.4)$$

From  $T_{hijk} l^k = 0$ , we have  $T_{\alpha\beta\gamma/\delta} = 0$ . Thus the surviving components  $T_{\alpha\beta\gamma\delta}$  are given by

$$T_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma;\delta}; \alpha, \beta, \gamma, \delta = 2, 3, 4, 5, 6 \quad (3.5)$$

If  $T_{\alpha\beta;\gamma}$  the scalar components of  $LT_j^i/k$ , i.e

$$LT_j^i/k = T_{\alpha\beta;\gamma} e_\alpha^i e_\beta)_j e_\gamma)_k, \quad (3.6)$$

then

$$T_{\alpha\beta;\gamma} = L(\partial_k T_{\alpha\beta}) e_\gamma^k + T_{\mu\beta} V_{\mu)\alpha\beta} + T_{\alpha\mu} V_{\mu)\beta\gamma} \quad (3.7)$$

Similarly, taking  $h$ - covariant differentiation of  $T_j^i$  with respect to  $k$  we get

$$T_{j/k}^i = (\delta_k T_{\alpha\beta} e_\alpha^i e_\beta)_j + T_{\alpha\beta} e_{\alpha)/k}^i e_\beta)_j + T_{\alpha\beta} e_\alpha^i e_\beta)_j / k \quad (3.8)$$

where  $\delta_k = \partial_k G_k^r \partial_r$ . If  $T_{\alpha\beta,\gamma}$  are scalar components of  $T_{j/k}^i$ , i.e

$$T_{j/k}^i = T_{\alpha\beta,\gamma} e_\alpha^i e_\beta^j, \quad (3.9)$$

then we obtain

$$T_{\alpha\beta,\gamma} = (\partial_k T_{\alpha\beta}) e_\gamma^k + T_{\mu\beta} H_{\mu\alpha\gamma} + T_{\alpha\mu} H_{\mu\beta\gamma} \quad (3.10)$$

The scalar components  $T_{\alpha\beta;\gamma}$  and  $T_{\alpha\beta,\gamma}$  are respectively called  $v$ - and  $h$ -scalar derivatives of scalar components  $T_{\alpha\beta}$  of  $T$ . By using (2.13) the explicit form of  $C_{\alpha\beta\gamma,\delta}$  are obtained as:

$$\left\{ \begin{array}{l} C_{222,\delta} = A_{,\delta} + 3(B + A' + E' + I')h_j + 3(C + G + F' + J')k_j + \\ 3(D + H + L + K')p_j + 3(E + I + M + C')j_j, \\ C_{223,\delta} = -(B + A' + E' + I')_{,\delta} - 2H'k_j + (C + G + F' + J')q_j - 2J'h_j - \\ 2I'p_j + (D + H + L + K')q_j - 2J'j'_j + (E + I + M + C')k'_j, \\ C_{224,\delta} = -(C + G + F' + J')_{,\delta} - 2H'h_j + (B + A' + \\ E' + I')j_j - \\ 2Ck_j - 2K'p_j + (D + H + L + K')h'_j - 2L'j'_j + (E + I + M + C')p'_j, \\ C_{225,\delta} = -(D + H + L + K')_{,\delta} - 2I'h_j - (B + A' + E' + I')q_j - 2K'k_j - 2Dp_j - (C + G + F' + \\ J')h'_j - 2M'j'_j + (E + I + M + C')q'_j, \\ C_{226,\delta} = -(E + I + M + C')_{,\delta} - 2J'h_j - (B + A' + E' + I')k'_j - 2L'k_j - 2M'p_j - (C + G + F' + \\ J')p'_j - 2Ej'_j - (D + H + L + K')q'_j, \\ C_{233,\delta} = B_{,\delta} - Fh_j - 2H'j_j - Jk_j - Np_j - 2I'q_j - D'j'_j - 2J'k'_j, \\ C_{244,\delta} = C_{,\delta} - Gh_j - 2H'j_j - Kk_j - A'p_j - 2K'h'_j - E'j'_j - 2L'p'_j, \\ C_{255,\delta} = D_{,\delta} - Hh_j - 2I'q_j - Lk_j - B'p_j - 2K'h'_j - F'j'_j - 2M'p'_j, \\ C_{266,\delta} = E_{,\delta} - Ih_j + 2J'k'_j - Mk_j - C'p_j + 2M'q'_j - G'j'_j + 2L'p'_j, \\ C_{333,\delta} = F_{,\delta} - 3Jj_j - 3Hq_j - 3D'p'_j, \\ C_{344,\delta} = G_{,\delta} + 2Jj_j - Kj_j - 3A'q_j - 2N'h'_j - E'j'_j - 2A''p'_j, \\ C_{355,\delta} = H_{,\delta} + 2Nq_j - Lj_j - B'q_j - 2N'h'_j - F'k'_j - 2B''q'_j, \\ C_{366,\delta} = I_{,\delta} + 2D'k'_j - Mj_j - B'q_j - 2A''p'_j - C'q'_j + 2B''q'_j + G'q'_j, \\ C_{433,\delta} = J_{,\delta} - Nh'_j + Fj_j - 2Gj_j - 2N'q_j - 2A''k'_j - D'p'_j, \\ C_{444,\delta} = K_{,\delta} + 3Gj_j - 3A'h'_j - 3E'p'_j, \\ C_{455,\delta} = L_{,\delta} + Hj_j - 2N'q_j - 2A'h'_j - B'p'_j - F'k'_j - 2C''q'_j, \\ C_{466,\delta} = M_{,\delta} + Ij_j - G'p'_j + 2A''k'_j + 2E'p'_j - C'h'_j + 2C''q'_j, \\ C_{533,\delta} = N_{,\delta} + Fq_j - Jh'_j - 2N'j_j - 2Hq_j - D'q'_j - 2B''k'_j, \\ C_{544,\delta} = A'_{,\delta} + Gq_j - 2N'j_j + Kh'_j - 2Lh'_j - E'q'_j - 2C''p'_j, \\ C_{555,\delta} = B'_{,\delta} + 3Hq_j - 3Lh'_j - 3F'q'_j, \end{array} \right. \quad (3.11)$$

$$\begin{cases}
C_{566,\delta} = C'_{,\delta} + Iq_j - 2B''k'_j + Mh'_j - G'q'_j - 2F''q'_j + 2C''p'_j, \\
C_{633,\delta} = D'_{,\delta} + Fk'_j + Jp'_j - 2A''j_j + Nq'_j - 2B''q_j - 2Ik'_j, \\
C_{644,\delta} = E'_{,\delta} + Gk'_j + Kp'_j + 2A''j_j + A'q'_j - 2C''h'_j - 2Mp'_j, \\
C_{655,\delta} = F'_{,\delta} + Hk'_j + Lp'_j + 2B''q_j + B'q'_j + 2C''h'_j - 2C'q'_j, \\
C_{666,\delta} = G'_{,\delta} + 3Ik'_j + 3Mp'_j + 3C'q'_j, \\
C_{234,\delta} = H'_{,\delta} - Jh_j + Bj_j - Gk_j - Cj_j - N'p_j - K'q_j + Ih'_j - 2A''j'_j - L'k'_j - J'q'_j, \\
C_{235,\delta} = I'_{,\delta} - Nh_j + Bq_j - Hp_j - K'j_j - N'k_j - Dq_j + H'h'_j - B''j'_j - M'k'_j - J'q'_j, \\
C_{236,\delta} = J'_{,\delta} - D'h_j + Bk'_j - A''k_j - L'j_j - H'p'_j - B''p_j - M'q_j - I'q'_j - Ij'_j - Ek'_j, \\
C_{245,\delta} = K'_{,\delta} - N'h_j - A'k_j - I'j_j - Ch'_j - Lp_j + H'q_j - Dh'_j - C''j'_j - M'p'_j - L'q'_j, \\
C_{246,\delta} = L'_{,\delta} - A''h_j + H'k'_j + J'j_j + Cp'_j - E'k_j - C''p_j - M'h'_j - Mj'_j - Ep'_j + K'q'_j, \\
C_{256,\delta} = M'_{,\delta} - B''h_j + I'k'_j + J'q_j + K'p'_j - C''k_j - L'h'_j - C'j'_j + Dq_j - Eq'_j - F'p_j, \\
C_{345,\delta} = N'_{,\delta} + Nj_j - A'j_j + Jq_j + Gh'_j - Lq_j - Hh'_j - C''k'_j + B''q'_j - A''q'_j, \\
C_{346,\delta} = A'_{,\delta} + D'j_j + Jk'_j - E'j_j - C''q_j + Gp'_j - B''h'_j - Mk'_j + N'q'_j - Ip'_j, \\
C_{356,\delta} = B'_{,\delta} + D'q_j + Nk'_j - A''j_j - B''q_j + N'p'_j + A''h'_j - Ik'_j + Hq'_j - Iq'_j, \\
C_{456,\delta} = C'_{,\delta} + N'k'_j + B''j_j + A''q_j + A'p'_j + E'h'_j - F'h'_j + Lq'_j - Mq'_j - C'p'_j, \\
C_{1\beta\gamma,\delta} = 0
\end{cases}$$

where  $A_{,\delta} = (\partial_k A)e_{\gamma}^k$ . Hence, we have the following result.

**Theorem 3.1.** *In a six dimensional Finsler space the  $h$ -covariant derivatives in terms of main scalars satisfying  $T$ -condition is given by equation (3.11).*

Similarly, using (2.15) the explicit form of  $C_{\alpha\beta\gamma;\delta}$  are obtained as follows:

$$\begin{cases}
C_{222;\delta} = A_{,\delta} + 3(B + A' + E' + I')u_j + 3(C + G + F' + J')w_j + \\
3(D + H + L + K')r_j + 3(E + I + M + C')v_j, \\
C_{223;\delta} = -(B + A' + E' + I')_{,\delta} - 2H'w_j + (C + G + F' + J')s_j - 2J'u_j - 2I'r_j + \\
(D + H + L + K')s_j - 2J'v'_j + (E + I + M + C')w'_j, \\
C_{224;\delta} = -(C + G + F' + J')_{,\delta} - 2H'u_j + (B + A' + E' + I')v_j - 2Cw_j - 2K'r_j + \\
(D + H + L + K')u'_j - 2L'v'_j + (E + I + M + C')r'_j, \\
C_{225;\delta} = -(D + H + L + K')_{,\delta} - 2I'u_j - (B + A' + E' + I')s_j - 2K'w_j - 2Dr_j - \\
(C + G + F' + J')u'_j - 2M'v'_j + (E + I + M + C')s'_j, \\
C_{226;\delta} = -(E + I + M + C')_{,\delta} - 2J'u_j - (B + A' + E' + I')w'_j - 2L'w_j - 2M'r_j - \\
(C + G + F' + J')r'_j - 2Ev'_j - (D + H + L + K')s'_j, \\
C_{233;\delta} = B_{,\delta} - Fu_j - 2H'v_j - Jw_j - Nr_j - 2I's_j - D'v'_j - 2J'w'_j, \\
C_{244;\delta} = C_{,\delta} - Gu_j - 2H'v_j - Kw_j - A'r_j - 2K'u'_j - E'v'_j - 2L'r'_j,
\end{cases} \quad (3.12)$$

$$\begin{cases}
C_{255;\delta} = D_{;\delta} - Hu_j - 2I' s_j - Lw_j - B' r_j - 2K' u'_j - F' v'_j - 2M' r'_j, \\
C_{266;\delta} = E_{;\delta} - Iu_j + 2J' w'_j - Mw_j - C' r_j + 2M' s'_j - G' v'_j + 2L' r'_j, \\
C_{333;\delta} = F_{;\delta} - 3Jv_j - 3Hs_j - 3D' r'_j, \\
C_{344;\delta} = G_{;\delta} + 2Jv_j - Kv_j - 3A' s_j - 2N' u'_j - E' v'_j - 2A'' r'_j, \\
C_{355;\delta} = H_{;\delta} + 2Ns_j - Lv_j - B' s_j - 2N' u'_j - F' w'_j - 2B'' s'_j, \\
C_{366;\delta} = I_{;\delta} + 2D' w'_j - Mv_j - B' s_j - 2A'' r'_j - C' s'_j + 2B'' s'_j + G' s'_j, \\
C_{433;\delta} = J_{;\delta} - Nu'_j + Fv_j - 2Gv_j - 2N' s_j - 2A'' w'_j - D' r'_j, \\
C_{444;\delta} = K_{;\delta} + 3Gv_j - 3A' u'_j - 3E' r'_j, \\
C_{455;\delta} = L_{;\delta} + Hv_j - 2N' s_j - 2A' u'_j - B' r'_j - F' w'_j - 2C'' s'_j, \\
C_{466;\delta} = M_{;\delta} + Iv_j - G' r'_j + 2A'' w'_j + 2E' r'_j - C' u'_j + 2C'' s'_j, \\
C_{533;\delta} = N_{;\delta} + Fs_j - Ju'_j - 2N' v_j - 2Hs_j - D' s'_j - 2B'' w'_j, \\
C_{544;\delta} = A_{;\delta} + Gs_j - 2N' v_j + Ku'_j - 2Lu'_j - E' s'_j - 2C'' r'_j, \\
C_{555;\delta} = B_{;\delta} + 3Hs_j - 3Lu'_j - 3F' s'_j, \\
C_{566;\delta} = C_{;\delta} + Is_j - 2B'' w'_j + Mu'_j - G' s'_j - 2F'' s'_j + 2C'' r'_j, \\
C_{633;\delta} = D'_{;\delta} + Fw'_j + Jr'_j - 2A'' v_j + Ns'_j - 2B'' s_j - 2Iw'_j, \\
C_{644;\delta} = E'_{;\delta} + Gw'_j + Kr'_j + 2A'' v_j + A' s'_j - 2C'' u'_j - 2Mr'_j, \\
C_{655;\delta} = F'_{;\delta} + Hw'_j + Lr'_j + 2B'' s_j + B' s'_j + 2C'' u'_j - 2C' s'_j, \\
C_{666;\delta} = G'_{;\delta} + 3Iw'_j + 3Mr'_j + 3C' s'_j, \\
C_{234;\delta} = H'_{;\delta} - Ju_j + Bv_j - Gw_j - Cv_j - N' r_j - K' s_j + Iu'_j - 2A'' v'_j - L' w'_j - J' s'_j, \\
C_{235;\delta} = I'_{;\delta} - Nu_j + Bs_j - Hr_j - K' v_j - N' w_j - Ds_j + H' u'_j - B'' v'_j - M' w'_j - J' s'_j, \\
C_{236;\delta} = J'_{;\delta} - D' u_j + Bw'_j - A'' w_j - L' v_j - H' r'_j - B'' r_j - M' s_j - I' s'_j - Iv_j - Ew'_j, \\
C_{245;\delta} = K'_{;\delta} - N' u_j - A' w_j - I' v_j - Cu'_j - Lr_j + H' s_j - Du'_j - C'' v'_j - M' r'_j - L' s'_j, \\
C_{246;\delta} = L'_{;\delta} - A'' u_j + H' w'_j + J' v_j + Cr'_j - E' w_j - C'' r_j - M' u'_j - Mv'_j - Er'_j + K' s'_j, \\
C_{256;\delta} = M'_{;\delta} - B'' u_j + I' w'_j + J' s_j + K' r'_j - C'' w_j - L' u'_j - C' v'_j + Ds'_j - Es'_j - F' r_j, \\
C_{345;\delta} = N'_{;\delta} + Nv_j - A' v_j + Js_j + Gu'_j - Ls_j - Hu'_j - C'' w'_j + B'' s'_j - A'' s'_j, \\
C_{346;\delta} = A''_{;\delta} + D' v_j + Jw'_j - E' v_j - C'' s_j + Gr'_j - B'' u'_j - Mw'_j + N' s'_j - Ir'_j, \\
C_{356;\delta} = B''_{;\delta} + D' s_j + Nw'_j - A'' v_j - B'' s_j + N' r'_j + A'' u'_j - Iw'_j + Hs'_j - Is'_j, \\
C_{456;\delta} = C''_{;\delta} + N' w'_j + B'' v_j + A'' s_j + A' r'_j + E' u'_j - F' u'_j + Ls'_j - Ms'_j - C' r'_j, \\
C_{1\beta\gamma;\delta} = -C_{\beta\gamma\delta},
\end{cases}$$

where  $A_{;\delta} = (\partial_k A) e^k_{\gamma}$ . Hence, we obtain the following result.

**Theorem 3.2.** *In a six dimensional Finsler space the  $v$ -covariant derivatives in terms of main scalars satisfying  $T$ -conditions is given by equation (3.12).*



#### 4. V-Curvature Tensor

M. Matsumoto [3] defined  $V$ -curvature tensor as

$$S_{hijk} = C_{hk}^r C_{ijr} - C_{hj}^r C_{ikr} \quad (4.1)$$

The scalar components of  $S_{\alpha\beta\gamma\delta}$  of  $L^2 S_{hijk}$  are given by [6, 7]

$$L^2 S_{hijk} = S_{\alpha\beta\gamma\delta} e_{\alpha} h e_{\alpha} i e_{\gamma} j e_{\delta} k \quad (4.2)$$

In view of equation (4.1) and (4.2) we have obtained 54 independent components in terms of main scalars. Since  $S_{hijk}$  is skew symmetric in  $h$  and  $i$  as well as in  $j$  and  $k$  and  $S_{0ijk} = S_{hi0k} = 0$ , the surviving independent components of  $S_{\alpha\beta\gamma\delta}$  are 54, which are obtained as:

$$\left\{ \begin{array}{l} S_{2323} = C_{23\theta} C_{\theta 32} - C_{22\theta} C_{\theta 33} \\ = 2B^2 + A'^2 + E'^2 + H'^2 + 2I'^2 + J'^2 + 2BA' + 2BE' + 2BI' \\ + 2A'E' + 2A'I' + 2E'I' - AB + FB + FA' + FE' + FI' + CJ \\ + JF' + JJ' + DN + HN + LN + NK' + ED' + ID' + MD' + C'D', \\ S_{2324} = C_{24\theta} C_{\theta 32} - C_{22\theta} C_{\theta 34} \\ = (C + G + F' + J')(B + A' + E' + I') - AH' \\ + H'B + (B + A' + E' + I')J + CH' + (C + G + I' + J')G \\ + K'I' + (D + H + L + K')N' + L'J' + (E + I + M + C')A'', \\ S_{2325} = C_{23\theta} C_{\theta 32} - C_{22\theta} C_{\theta 35} \\ = (D + H + L + K')(B + A' + E' + I') - AI' + I'B \\ + (B + A' + E' + I')N + K'H' + (C + G + F' + J')N' + DI' \\ + (D + H + L + K')H + M'J' + (E + I + M + C')A'', \\ S_{2326} = C_{26\theta} C_{\theta 32} - C_{22\theta} C_{\theta 36} \\ = (B + A' + E' + I')(E + I + M + C') - AJ' + BJ' \\ + (B + A' + E' + I')D' + L'H' + 9C + G + F' + J')A'' + M'I' \\ + 9D + H + L + K')B'' + EJ' + (E + I + M + C')I, \\ S_{2334} = C_{24\theta} C_{\theta 33} - C_{23\theta} C_{\theta 34} \\ = -(C + G + F' + J')B + (B + A' + E' + I')H' + IH' - BJ \\ + CJ - GH' + NK' - I'N' + L'D' - J'A'', \\ S_{2335} = C_{25\theta} C_{\theta 33} - C_{23\theta} C_{\theta 35} \\ = -(D + H + L + K')B + (B + A' + E' + I')I' + FI' - BN \\ - JK' - HI' + DN + M'D' + L'D' - J'B'', \\ S_{2336} = C_{26\theta} C_{\theta 33} - C_{23\theta} C_{\theta 36} \\ = -(E + I + M + C')B + (B + A' + E' + I')J' + FJ' - BD' \\ + JL' - H'A'' + M'N - I'B'' + ED' - J'I, \end{array} \right.$$

$$\begin{aligned}
S_{2424} &= C_{246}C_{\theta 42} - C_{22\theta}C_{\theta 44} \\
&= (C + G + F' + J')^2 - AC + H'^2 + (B + A' + E' + I')G + C^2 \\
&\quad + (C + G + F' + J')K + K'^2 + (D + H + L + K')A' + L'^2 \\
&\quad + (E + I + M + C')E', \\
S_{2425} &= C_{256}C_{\theta 42} - C_{22\theta}C_{\theta 45} \\
&= (D + H + L + K')(C + G + F' + J') - AK' + I'H' \\
&\quad + (B + A' + E' + I')N' + K'C + (C + G + F' + J')K' + DK' \\
&\quad + (D + H + L + K')L + M'L' + (E + I + M + C')C'', \\
S_{2426} &= C_{266}C_{\theta 42} - C_{22\theta}C_{\theta 46} \\
&= (E + I + M + C')(C + G + F' + J') - AL' + J'H' \\
&\quad + (B + A' + E' + I')A'' + L'C + (C + G + F' + J')E' + M'K' \\
&\quad + (D + H + L + K')L' + EL' + (E + I + M + C')M, \\
S_{2345} &= C_{256}C_{\theta 34} - C_{24\theta}C_{\theta 35} \\
&= -(D + H + L + K')H' + (C + G + F' + J')I' + JI' - NH' \\
&\quad + GK' - CN' + DN' - K'H + A''M' - L'B'', \\
S_{2346} &= C_{266}C_{\theta 33} - C_{24\theta}C_{\theta 36} \\
&= (E + I + M + C')B + (C + G + F' + J')J' + FJ' \\
&\quad - J'D' - JL' - CA'' + NM' - B''K' + ED' - IL', \\
S_{2525} &= C_{256}C_{\theta 52} - C_{22\theta}C_{\theta 55} \\
&= (D + H + L + K')^2 - AD + I'^2 + (B + A' + E' + I')H \\
&\quad + K'^2 + (C + G + F' + J')L + D^2 + (D + H + L + K')B' \\
&\quad + M'^2 + (E + I + M + C')F', \\
S_{2526} &= C_{266}C_{\theta 52} - C_{22\theta}C_{\theta 56} \\
&= (D + H + L + K')^2 - AD + I'^2 + (B + A' + E' + I')H + K'^2 \\
&\quad + (C + G + F' + J')L + D^2 + (D + H + L + K')B' + M'^2 \\
&\quad + (E + I + M + C')F', \\
S_{2434} &= C_{246}C_{\theta 43} - C_{23\theta}C_{\theta 44} \\
&= (D + H + L + K')H' + (B + A' + E' + I')C + JH' \\
&\quad - BG + CG - H'K + K'N' - A'I' + L'A'' - J'E', \\
S_{2435} &= C_{256}C_{\theta 43} - C_{23\theta}C_{\theta 45} \\
&= -(D + H + L + K')H' + (B + A' + E' + I')K' + JI' \\
&\quad - BN' + K'G - H'A' + DN' - I'L + M'A'' - J'C'',
\end{aligned}$$

$$\begin{aligned}
S_{2436} &= C_{266}C_{\theta 43} - C_{23\theta}C_{\theta 46} \\
&= -(E + I + M + C')H' + (B + A' + E' + I')L' + JJ' \\
&\quad - BA'' + L'G - H'E' + M'N' - I'C'' + EA'' - J'M, \\
S_{2445} &= C_{256}C_{\theta 44} - C_{24\theta}C_{\theta 45} \\
&= -(D + H + L + K')C + (C + G + F' + J')K' + I'G \\
&\quad - H'N' + K'K - CA' + DA' - K'L + E'M' - L'C'', \\
S_{2446} &= C_{266}C_{\theta 44} - C_{24\theta}C_{\theta 46} \\
&= -(E + I + M + C')C + (C + G + F' + J')L' + J'G \\
&\quad - H'A'' + L'K - CE' + M'A' - K'C'' + E'E - L'M, \\
S_{2456} &= C_{266}C_{\theta 45} - C_{25\theta}C_{\theta 46} \\
&= -(E + I + M + C')K' + (D + H + L + K')L' + J'N' \\
&\quad - I'A''L'A' - K'E' + M'L - DC'' + C''E - M'M, \\
S_{2534} &= C_{246}C_{\theta 53} - C_{23\theta}C_{\theta 54} \\
&= -(C + G + F' + J')I' + (B + A' + E' + I')K' + H'N \\
&\quad - BN' + CN' - H'A' + K'N - I'A' + L'M' - J'C'', \\
S_{2535} &= C_{256}C_{\theta 53} - C_{23\theta}C_{\theta 55} \\
&= -(D + H + L + K')I' + (B + A' + E' + I')D + I'N \\
&\quad - BH - K'N' - H'L + DH - I'B' + M'B'', \\
S_{2536} &= C_{266}C_{\theta 53} - C_{23\theta}C_{\theta 56} \\
&= -(E + I + M + C')I' + (B + A' + E' + I')M' + J'N \\
&\quad - BB' - L'N' - H'C'' + M'H - I'F' + EB'' - J'C', \\
S_{2545} &= C_{256}C_{\theta 54} - C_{24\theta}C_{\theta 55} \\
&= -(D + H + L + K')K' + (C + G + F' + J')D + I'N' \\
&\quad - HH' + K'A' - CL + DL - K'B' + M'C'' - L'F', \\
S_{2546} &= C_{266}C_{\theta 54} - C_{24\theta}C_{\theta 56} \\
&= -(E + I + M + C')K' + (C + G + F' + J')M' + J'N' \\
&\quad - H'B'' + L'A' - CC'' + M'L - K'F' + EC'' - L'C', \\
S_{2556} &= C_{266}C_{\theta 55} - C_{25\theta}C_{\theta 56} \\
&= -(E + I + M + C')D + (D + H + L + K')M' + J'H \\
&\quad - I'B'' + L'L - K'C'' + L'B' - DF' + EF' - M'C', \\
S_{2626} &= C_{266}C_{\theta 62} - C_{22\theta}C_{\theta 66} \\
&= (E + I + M + C')^2 - AE + J'^2 + (B + A' + E' + I')I \\
&\quad + L'^2 + (C + G + F' + J')M + M'^2 + (D + H + L + K')C' \\
&\quad + E^2 + (E + I + M + C')G', \\
S_{2634} &= C_{246}C_{\theta 63} - C_{23\theta}C_{\theta 64} \\
&= -(C + G + I' + J')J' + (B + A' + E' + I')L' + H'D' \\
&\quad - BA'' + CA'' - H'E' + K'B'' - I'C'' + IL' - J'M, \\
S_{2635} &= C_{256}C_{\theta 63} - C_{23\theta}C_{\theta 65} \\
&= -(D + H + L + K')J' + (B + A' + E' + I')M' + I'D' \\
&\quad - BB'' + K'A'' - H'C'' + DB'' - I'F' + IM' - J'C',
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
S_{2636} &= C_{26\theta}C_{\theta 63} - C_{23\theta}C_{\theta 66} \\
&= -(E + I + M + C')J' + (B + A' + E' + I')E + J'D' \\
&\quad - BI + L'A'' - MH' + M'B'' - I'C' + EI - J'G', \\
S_{2645} &= C_{25\theta}C_{\theta 64} - C_{24\theta}C_{\theta 65} \\
&= -(D + H + L + K')L' + (C + G + F' + J')M' + I'A'' \\
&\quad - H'B'' + K'E' - CC'' + DC'' - K'F' + M'M - L'C', \\
S_{2646} &= C_{26\theta}C_{\theta 64} - C_{24\theta}C_{\theta 66} \\
&= -(E + I + M + C')L' + (C + G + F' + J')E + J'A'' \\
&\quad - H'I + L'E' - CM + J'C'' - K'C' + EM - L'G', \\
S_{2656} &= C_{26\theta}C_{\theta 65} - C_{25\theta}C_{\theta 66} \\
&= -(E + I + M + C')M' + (D + H + L + K')E' + J'B'' \\
&\quad - I'I + L'C'' - K'M + M'F' - DC' + EC' - M'G', \\
S_{3434} &= C_{34\theta}C_{\theta 43} - C_{33\theta}C_{\theta 44} \\
&= H'^2 - BC + J^2 - FG + G^2 - JG + N'^2 - NA' + A''^2 - D'E', \\
S_{3446} &= C_{36\theta}C_{\theta 44} - C_{34\theta}C_{\theta 46} \\
&= J'C - H'L' + D'G - JA'' + A'K - GE' + A'B'' - N'C'' \\
&\quad + IE' - A''M, \\
S_{3456} &= C_{36\theta}C_{\theta 45} - C_{35\theta}C_{\theta 46} \\
&= J'K' - I'L' + D'N' - NA'' + A''A - N'E' + B''L \\
&\quad - HC'' + IC'' - B''M, \\
S_{3535} &= C_{35\theta}C_{\theta 53} - C_{33\theta}C_{\theta 55} \\
&= I'^2 - BD + N^2 - FH + N'^2 - JL + H^2 - NB' + B''^2 - D'F', \\
S_{3536} &= C_{36\theta}C_{\theta 53} - C_{33\theta}C_{\theta 56} \\
&= J'I' - BM' + D'N - FB'' + A''N' - JC'' + B''H \\
&\quad - NF' + IB'' - D'C', \\
S_{3435} &= C_{35\theta}C_{\theta 43} - C_{33\theta}C_{\theta 45} \\
&= I'H' - BK' + NJ - FN' + N'G - JA' + HN' \\
&\quad - NL + B''A'' - D'C'', \\
S_{3445} &= C_{35\theta}C_{\theta 44} - C_{34\theta}C_{\theta 45} \\
&= I'C - H'K' + NG - JN' + N'K - GA' + HA' \\
&\quad - N'L + B''E' - A''C'', \\
S_{3436} &= C_{36\theta}C_{\theta 43} - C_{33\theta}C_{\theta 46} \\
&= J'H' - BL' + D'J - FA'' + A''G - JE' + B''N' \\
&\quad - NC'' + IA'' - D'M, \\
S_{3545} &= C_{35\theta}C_{\theta 54} - C_{34\theta}C_{\theta 55} \\
&= I'k' - H'D + N'N - JH + A'N' - GL \\
&\quad - HL - N'B' + B''C'' - A''F', \\
S_{3546} &= C_{36\theta}C_{\theta 54} - C_{34\theta}C_{\theta 56} \\
&= J'H' - H'M' + D'J - JB'' + A''G - GC'' \\
&\quad + B''N' - N'F' + IA'' - A''C', \\
S_{3556} &= C_{36\theta}C_{\theta 55} - C_{35\theta}C_{\theta 56} \\
&= J'D - I'M' + D'H - NB'' + A''L - N'C'' \\
&\quad + B''B' - HF' + IF' - B''C',
\end{aligned}$$

$$\left\{ \begin{aligned}
S_{3636} &= C_{36\theta}C_{\theta63} - C_{33\theta}C_{\theta66} \\
&= J'^2 - BE + D'^2 - FI + A''^2 - JM + B'^2 - NC' + I^2 - D'G', \\
S_{3645} &= C_{35\theta}C_{\theta64} - C_{34\theta}C_{\theta65} \\
&= I'L' - H'M' + NA'' - JB'' + N'E' - GC'' \\
&+ HC'' - N'F' + B''M - A''C', \\
S_{3646} &= C_{36\theta}C_{\theta64} - C_{34\theta}C_{\theta66} \\
&= J'L' - H'E + D'A'' - JE + A''E' - GM + B''C'' - N'C' + IM - A''G', \\
S_{3656} &= C_{36\theta}C_{\theta65} - C_{35\theta}C_{\theta66} \\
&= J'M' - I'E + D'B'' - NI + A''B'' - N'M \\
&+ B''F' - HC' + IC' - B''G', \\
S_{4545} &= C_{45\theta}C_{\theta54} - C_{44\theta}C_{\theta55} \\
&= K'^2 - CD + N'^2 - GH + A'^2 - KL + L^2 - A'B' + C''^2 - E'F', \\
S_{4546} &= C_{46\theta}C_{\theta54} - C_{44\theta}C_{\theta56} \\
&= L'K' - CM' + N'A'' - GB'' + C''L - A'F' + MC'' - E'C', \\
S_{4556} &= C_{46\theta}C_{\theta55} - C_{45\theta}C_{\theta56} \\
&= L'D - K'M' + A''H - N'B'' + E'L - A'C'' \\
&+ C''B' - LF' + MF' - C''C', \\
S_{4646} &= C_{46\theta}C_{\theta64} - C_{44\theta}C_{\theta66} \\
&= L'^2 - CE + A'^2 - GI + E'^2 - KM + C''^2 - A'C' + M^2 - E'G', \\
S_{4656} &= C_{46\theta}C_{\theta65} - C_{45\theta}C_{\theta66} \\
&= L'M' - K'E + A''B'' - N'I + E'C'' - A'M \\
&+ C''F' - LC' + MC' - C''G', \\
S_{5656} &= C_{56\theta}C_{\theta65} - C_{55\theta}C_{\theta66} \\
&= M'^2 - DE + B''^2 - HI + C''^2 - LM \\
&+ F'^2 - B'C' + C'^2 - F'G',
\end{aligned} \right.$$

**Theorem 4.1.** *In a six- dimensional Finsler space the V-curvature tensor in terms of main scalars are given by equation (4.3).*

## Conclusion

This research paper provides fundamental insights into the relationship between tensors and scalars of Finsler spaces, highlighting the intricate nature of this mathematical framework. Overall it offers a detailed exploration of tensors and their derivative in Finsler spaces.

## REFERENCES

1. A. Moor, *Uberdie Torsion-Und Krümmung invarianten der deridimensionalen Finsler-schen Raum*, Math. Nachr. 16(1957), 85-99.

2. M. Matsumoto and R. Miron, *On an invariant theory of Finsler Spaces*, Period. Math. Hunger, 8(1) (1977), 73-82.
3. M. Matsumoto, *Foundation of Finsler geometry and special Finsler spaces*, Kaiseisha Press, Shigaken, 1986.
4. M. K. Gupta and P. N. Pandey, *On S-3 like Four-dimensional Finsler spaces*, Acta Math. Aca. Paeda. Nyiregyhaziensis, 26(2010), 305-312.
5. M. Matsumoto and H. Shimada, *On Finsler spaces with curvature tensor  $P_{hijk}$  and  $S_{hijk}$  satisfying special conditions*, Rep. on Math. Phys., 12(1977), 77-87.
6. P. K. Dwivedi, S. C. Rastogi and A. K. Dwivedi, *Theory of six dimensional Finsler spaces in terms of scalars*, Mukt Shabd Journal, vol(9), issue-6, (2020), 2007-2022.
7. S. K. Tiwari and Anamika Rai, *On S-3 like Five-dimensional Finsler spaces*, The Scientific Temper, 12(2021), 59-64.

Received: 06.01.2024

Accepted: 14.02.2024