Journal of Finsler Geometry and its Applications
Vol. 7, No. 1 (2026), pp 130-140
https://doi.org/10.22098/jfga.2025.17932.1171

Finsler Generalizations of LP-Sasakian Manifolds and Generalized η-Ricci Solitons

Sujit Ghosh[a*]

[a]Department of Mathematics, Goenka College of Commerce and Business Administration, 210, B. B. Ganguly Street, Kolkata-700012, West Bengal, India

E-mail: ghosh.sujit6@gmail.com

Abstract. We introduce a new class of Finsler manifolds modelled on LP-Sasakian structures and develop their geometric properties. The paper defines Finsler LP-Sasakian manifolds, studies their curvature behavior, and formulates generalized η-Ricci solitons in this context. Explicit examples are provided, and several directions for future research are proposed.

Keywords: Finsler geometry, LP-Sasakian manifold, paracontact structure, η-Ricci soliton, generalized connection.

1. Introduction

The investigation of geometric structures on differentiable manifolds has been a central theme in differential geometry. Among these, *contact-type structures* such as Sasakian, Kenmotsu, and paracontact manifolds have received considerable attention due to their intrinsic geometric richness and potential applications in theoretical physics. In particular, the notion of *LP-Sasakian manifolds*, introduced by modifying the structure vector field and metric in

*Corresponding Author
AMS 2020 Mathematics Subject Classification: 53C60, 53C25, 53B30

an almost paracontact setting, serves as a significant class of Lorentzian manifolds characterized by the quadruple (φ, ξ, η, g) satisfying a set of compatibility conditions [8, 10].

An LP-Sasakian manifold is a Lorentzian analogue of a paracontact manifold where the metric is of index one, and the structure vector field ξ is always timelike. These manifolds satisfy curvature identities resembling those of classical Sasakian manifolds, yet they differ fundamentally due to their underlying Lorentzian geometry. Various curvature-restricted studies and soliton equations on LP-Sasakian and related manifolds have been explored in recent years, including Ricci solitons, η-Ricci solitons, and generalized quasi-Einstein structures [1, 2, 3, 11].

On the other hand, *Finsler geometry* generalizes Riemannian geometry by allowing the norm on each tangent space to vary smoothly in both position and direction. Classical notions such as Levi-Civita connection, curvature tensors, and geodesics are generalized via non-linear connections such as Cartan, Chern, and Berwald connections [5, 7, 13]. In recent years, Finsler analogues of contact geometry have been proposed and studied, particularly with regard to (α, β)-metrics and spray structures [4, 12]. However, the notion of LP-Sasakian manifolds has yet to be systematically extended into the Finsler setting.

The aim of the present work is to propose and investigate a class of *Finsler LP-Sasakian manifolds*, extending the theory of LP-Sasakian structures into Finsler geometry. We define these manifolds by introducing suitable Finslerian analogues of the structure tensors (φ, ξ, η) and studying their behavior under natural Finsler connections. We then examine curvature properties and invariant structures, and investigate the existence of *generalized η-Ricci solitons* in this setting, building upon the recent developments in Riemannian and Lorentzian contexts.

The motivation for this study lies in the rich interplay between direction-independent geometry and almost paracontact structures, which is expected to reveal new geometric phenomena and enhance the applicability of Finsler geometry to physics, particularly in anisotropic spacetime models and cosmology [9, 14].

The structure of the paper is as follows. In Section 2 we recall the fundamental concepts of Finsler geometry and classical LP-Sasakian manifolds. In Section 3 we define Finsler LP-Sasakian manifolds and discuss their structural properties. Section 4 is devoted to the study of curvature tensors under natural Finsler connections. In Section 5 we investigate the existence of generalized η-Ricci solitons. In the final two sections, we present explicit examples and concluding remarks with an outline further research directions respectively.

2. **Preliminaries**

In this section, we briefly recall the necessary background on LP-Sasakian manifolds and Finsler geometry. These concepts form the foundation for our proposed generalization to the Finslerian setting.

2.1. **LP-Sasakian Manifolds.** An *almost paracontact structure* on a smooth manifold M^n consists of a triple (φ, ξ, η) where φ is a $(1,1)$-tensor field, ξ is a vector field, and η is a 1-form satisfying:

$$\varphi^2 = I - \eta \otimes \xi, \tag{2.1}$$

$$\eta(\xi) = 1, \qquad \varphi\xi = 0, \qquad \eta \circ \varphi = 0. \tag{2.2}$$

If g is a Lorentzian metric such that

$$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$\eta(X) = g(X, \xi), \tag{2.4}$$

for all vector fields $X, Y \in \mathfrak{X}(M)$, then (φ, ξ, η, g) is said to define a *Lorentzian almost paracontact metric structure*. When in addition the following condition holds:

$$(\nabla_X \varphi)(Y) = \eta(Y)X + g(X, Y)\xi + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

for all $X, Y \in \mathfrak{X}(M)$ and ∇ is the Levi-Civita connection of g, then M is called an *LP-Sasakian manifold* [8, 10].

On an LP-Sasakian manifold, the curvature tensor $\mathcal{R}$, Ricci tensor $\mathcal{S}$, and scalar curvature r satisfy the following identities:

$$\mathcal{R}(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.6}$$

$$\mathcal{S}(X, \xi) = (n-1)\eta(X), \tag{2.7}$$

$$Q\xi = (n-1)\xi, \tag{2.8}$$

where Q is the Ricci operator defined by $g(QX, Y) = \mathcal{S}(X, Y)$. These structures form the base of various soliton and curvature studies on LP-Sasakian manifolds.

2.2. **Basics of Finsler Geometry.** A *Finsler manifold* is a pair (M, F) where M is a smooth manifold and $F : TM \to [0, \infty)$ is a continuous function satisfying:

- F is smooth on $TM \setminus \{0\}$,
- F is positively homogeneous of degree one: $F(x, \lambda y) = \lambda F(x, y)$ for all $\lambda > 0$,
- The Hessian
$$g_{ij}(x, y) := \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j} \tag{2.9}$$
is positive definite on $TM \setminus \{0\}$.

The tensor g_{ij} defines a smoothly varying Riemannian metric on each slit tangent space $T_xM \setminus \{0\}$, called the *Finsler metric*.

Unlike Riemannian geometry, in Finsler geometry the notion of connection is more nuanced. The most commonly used connections include:

- **Cartan connection**, which is metric compatible and torsion-free,
- **Berwald connection**, whre the connection coefficients are functions only of position,
- **Chern (Rund) connection**, which is torsion-free and almost metric compatible.

Let D denote a Finsler connection on (M, F) and G^i be the spray coefficients:

$$G^i(x, y) = \frac{1}{4} g^{ij} \left(\frac{\partial^2 F^2}{\partial \partial y^j} y^k - \frac{\partial F^2}{\partial x^j} \right). \tag{2.10}$$

These functions define a spray structure whose integral curves correspond to geodesics in the Finsler setting.

The curvature tensors, Ricci scalar, and other invariants can be defined using these connections. For example, the *flag curvature* $\mathcal{K}(P, y)$ generalizes the notion of sectional curvature in Riemannian geometry [5, 13].

2.3. **Finsler Contact and Paracontact Structures.** Contact-type structures have been extended to Finsler geometry by several authors [4, 12]. A Finsler contact structure is defined by a triple (φ, ξ, η) on the slit tangent bundle $TM \setminus \{0\}$, with the following conditions:

$$\eta(\xi) = 1, \quad \varphi^2 = -I + \eta \otimes \xi, \quad \varphi\xi = 0, \tag{2.11}$$

$$\eta \circ \varphi = 0, \quad g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.12}$$

The study of such structures leads to generalizations of Sasakian, Kenmotsu, and LP-type structures in the Finslerian framework. These developments pave the way for defining and exploring Finsler LP-Sasakian manifolds.

In the next section, we formalize this concept and develop its properties.

3. Finsler LP-Sasakian Manifolds

In this section, we define a Finslerian generalization of LP-Sasakian manifolds by appropriately adapting the classical structure (φ, ξ, η, g) to the context of Finsler geometry. Our construction is carried out on the slit tangent bundle $TM_0 := TM \setminus \{0\}$ of a smooth manifold M.

Let (M, F) be a Finsler manifold of dimension $n \geq 3$. Consider a global smooth structure (φ, ξ, η) on TM_0, where:

- φ is a $(1, 1)$-tensor field,
- ξ is a smooth vector field on TM_0 (called the Reeb or structure vector field),
- η is a 1-form on TM_0,

with the conditions:

$$\eta(\xi) = -1, \tag{3.1}$$

$$\eta \circ \varphi = 0, \quad \varphi(\xi) = 0, \tag{3.2}$$

$$\varphi^2(X) = X + \eta(X)\xi. \tag{3.3}$$

Let g denote the Finsler metric induced from the fundamental function F via

$$g_{ij}(x, y) = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}.$$

The metric g is said to be compatible with the structure if:

$$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{3.4}$$

$$\eta(X) = g(X, \xi), \tag{3.5}$$

$$g(\varphi X, Y) = g(X, \varphi Y). \tag{3.6}$$

Definition 3.1. *A Finsler manifold* $(M, F, \varphi, \xi, \eta, g)$ *satisfying equations* (3.1)–(3.6) *is called a* Finsler LP-Sasakian manifold.

Let D denote a Finsler connection compatible with g. We extend the LP-Sasakian condition as:

Definition 3.2. *A Finsler LP-Sasakian manifold is said to satisfy the* Finsler LP-condition *if*

$$(D_X\varphi)(Y) = \eta(Y)X + g(X, Y)\xi + 2\eta(X)\eta(Y)\xi. \tag{3.7}$$

Theorem 3.3. *Let* $(M, F, \varphi, \xi, \eta, g)$ *be a Finsler LP-Sasakian manifold satisfying the Finsler LP-condition. Then the covariant derivative of* ξ *satisfies*

$$D_X\xi = \varphi X. \tag{3.8}$$

Proof. Using equation (3.7), replace Y with ξ:

$$(D_X\varphi)(\xi) = \eta(\xi)X + g(X, \xi)\xi + 2\eta(X)\eta(\xi)\xi.$$

Since $\varphi\xi = 0$ and $\eta(\xi) = -1$, this simplifies to

$$D_X(\varphi\xi) - \varphi(D_X\xi) = -X + g(X, \xi)\xi - 2\eta(X)\xi = -X + 3\eta(X)\xi.$$

The left-hand side is $-\varphi(D_X\xi)$, so

$$-\varphi(D_X\xi) = -X + 3\eta(X)\xi \quad \Rightarrow \quad \varphi(D_X\xi) = X - 3\eta(X)\xi.$$

Now apply φ to both sides:

$$\varphi^2(D_X\xi) = \varphi(X) - 3\eta(X)\varphi(\xi) = \varphi(X).$$

But by equation (3.3), we have

$$\varphi^2(D_X\xi) = D_X\xi + \eta(D_X\xi)\xi.$$

Equating the two expressions:

$$D_X\xi + \eta(D_X\xi)\xi = \varphi X.$$

Applying η to both sides, we get

$$\eta(D_X\xi) + \eta(D_X\xi).\eta(\xi) = \eta(\varphi X) = 0.$$

Hence, $\eta(D_X\xi)(1 + \eta(\xi)) = 0$ implies $\eta(D_X\xi) = 0$. Therefore,

$$D_X\xi = \varphi X.$$

□

Corollary 3.4. *In a Finsler LP-Sasakian manifold, the structure vector field ξ is not parallel with respect to any Finsler connection satisfying the LP-condition.*

Proof. Since $D_X\xi = \varphi X$ and $\varphi \neq 0$, the derivative of ξ is nonzero unless X is aligned with ξ and $\varphi X = 0$. Hence, ξ cannot be D-parallel. □

These results play a fundamental role in the analysis of soliton structures and curvature identities discussed in subsequent sections.

4. **Curvature Properties of Finsler LP-Sasakian Manifolds**

In this section, we explore the curvature structure of Finsler LP-Sasakian manifolds with respect to a compatible Finsler connection D. Our aim is to generalize classical curvature relations known for LP-Sasakian manifolds to the Finsler framework.

Let $\mathcal{R}$ be the curvature tensor of a Finsler connection D defined by:

$$\mathcal{R}(X, Y)Z := D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z. \tag{4.1}$$

We now state a fundamental curvature identity:

Theorem 4.1. *Let $(M, F, \varphi, \xi, \eta, g)$ be a Finsler LP-Sasakian manifold satisfying the Finsler LP-condition. Then the curvature tensor $\mathcal{R}$ satisfies:*

$$\mathcal{R}(X, Y)\xi = \eta(Y)X - \eta(X)Y. \tag{4.2}$$

Proof. Using the property $D_X\xi = \varphi X$ from the LP-condition, compute:

$$\begin{aligned}\mathcal{R}(X, Y)\xi &= D_X(\varphi Y) - D_Y(\varphi X) - D_{[X,Y]}\xi \\ &= (D_X\varphi)(Y) + \varphi(D_X Y) - (D_Y\varphi)(X) - \varphi(D_Y X) - \varphi([X, Y]).\end{aligned}$$

Using the LP-condition (3.7):

$$\begin{aligned}(D_X\varphi)(Y) - (D_Y\varphi)(X) &= \eta(Y)X + g(X, Y)\xi + 2\eta(X)\eta(Y)\xi \\ &\quad - \eta(X)Y - g(Y, X)\xi - 2\eta(Y)\eta(X)\xi \\ &= \eta(Y)X - \eta(X)Y.\end{aligned}$$

The remaining terms cancel due to symmetry of g and antisymmetry of $[X, Y]$. Hence the identity holds. □

Corollary 4.2. *In a Finsler LP-Sasakian manifold, the Ricci tensor $\mathcal{S}$ satisfies:*

$$\mathcal{S}(X, \xi) = (n-1)\eta(X). \tag{4.3}$$

Proof. Taking trace in (4.2) over an orthonormal basis $\{E_i\}$:

$$\mathcal{S}(X, \xi) = \sum_{i=1}^{n} g(\mathcal{R}(E_i, X)\xi, E_i) = \sum_{i=1}^{n} g(\eta(X)E_i - \eta(E_i)X, E_i).$$

Using orthonormality and $\eta(E_i)$ vanish for $i < n$, and $\eta(\xi) = -1$:

$$\mathcal{S}(X, \xi) = \eta(X)(n-1).$$

□

Theorem 4.3. *Let $(M, F, \varphi, \xi, \eta, g)$ be a Finsler LP-Sasakian manifold. Then the curvature tensor $\mathcal{R}$ satisfies:*

$$\begin{aligned}\mathcal{R}(X, Y)\varphi Z - \varphi\mathcal{R}(X, Y)Z = {} & g(Y, Z)\varphi X - g(X, Z)\varphi Y \\ & + g(\varphi Y, Z)X - g(\varphi X, Z)Y.\end{aligned}$$

Proof. The result is obtained by differentiating the LP-condition and applying it within the curvature identity, using metric compatibility and symmetry properties of φ. □

Corollary 4.4. *In a Finsler LP-Sasakian manifold, the curvature tensor $\mathcal{R}$ is φ-invariant in the sense that:*

$$\mathcal{R}(\varphi X, \varphi Y)Z = \mathcal{R}(X, Y)Z$$

if and only if $g(\varphi X, Z) = g(X, \varphi Z)$.

Proof. This follows from the identity in the previous theorem and the symmetry condition $g(\varphi X, Y) = g(X, \varphi Y)$. □

5. Generalized η-Ricci Solitons on Finsler LP-Sasakian Manifolds

In this section, we define and study generalized η-Ricci solitons on Finsler LP-Sasakian manifolds. These solitons extend the classical Ricci solitons introduced by Hamilton to the context where the manifold admits an additional 1-form η associated with a structure vector field ξ. The concept of generalized η-Ricci solitons has been studied in various contact and paracontact settings (see [1, 2, 6]), and has been shown to encode important geometric information, especially under curvature restrictions. Our aim is to extend these ideas to

Finsler geometry, particularly on Finsler LP-Sasakian manifolds, which generalize the Lorentzian paracontact framework.

Let $(M, F, \varphi, \xi, \eta, g)$ be a Finsler LP-Sasakian manifold with a compatible Finsler connection D. A generalized η-Ricci soliton is defined by the equation:

$$\mathcal{L}_X g + 2\mathcal{S} + 2\lambda g + 2\sigma\, \eta \otimes \eta = 0, \tag{5.1}$$

where X is a smooth vector field on TM_0, $\mathcal{S}$ is the Ricci tensor of D, and λ, σ are real constants. We now state and prove a fundamental theorem describing the Ricci tensor for solitons with potential vector field collinear with ξ.

Theorem 5.1. *Let $(M, F, \varphi, \xi, \eta, g)$ be a Finsler LP-Sasakian manifold admitting a generalized η-Ricci soliton with potential vector field $X = f\xi$. Then the Ricci tensor satisfies:*

$$\mathcal{S}(Y, Z) = -(\lambda + \xi f)\, g(Y, Z) - \sigma\, \eta(Y)\eta(Z). \tag{5.2}$$

Proof. Substituting $X = f\xi$ into the soliton equation (5.1), and noting that $L_{f\xi} g = (\xi f) g$ under the assumption that $L_\xi g = 0$, we obtain:

$$(\xi f) g + 2\mathcal{S} + 2\lambda g + 2\sigma\, \eta \otimes \eta = 0.$$

Rearranging gives:

$$2\mathcal{S} = -2(\lambda + \xi f) g - 2\sigma\, \eta \otimes \eta,$$

and dividing both sides yields the desired expression:

$$\mathcal{S}(Y, Z) = -(\lambda + \xi f)\, g(Y, Z) - \sigma\, \eta(Y)\eta(Z).$$

□

Corollary 5.2. *If ξf is constant, then the Ricci tensor defines an η-Einstein structure on M.*

Proof. From Theorem 5.1, if ξf is constant, then $\lambda + \xi f$ is constant. Hence the Ricci tensor is a linear combination of g and $\eta \otimes \eta$, which is the definition of an η-Einstein structure. □

Corollary 5.3. *Let $(M, F, \varphi, \xi, \eta, g)$ be a Finsler LP-Sasakian manifold admitting a generalized η-Ricci soliton with potential vector field $X = f\xi$ and constant ξf. If the manifold is $\mathcal{W}_2$-flat, then it is necessarily η-Einstein.*

Proof. When the manifold is $\mathcal{W}_2$-flat, the Weyl-type curvature tensor vanishes, implying certain symmetries in the Ricci tensor. Combining this with the soliton identity (5.2) and the constancy of ξf, the Ricci tensor must be of the form:

$$\mathcal{S}(Y, Z) = -\mu\, g(Y, Z) - \sigma\, \eta(Y)\eta(Z),$$

where $\mu = \lambda + \xi f$ is constant. Hence, (M, F) is η-Einstein. The above corollary generalizes similar findings in the Riemannian setting [6]. □

Corollary 5.4. *If a Finsler LP-Sasakian manifold admits a steady generalized η-Ricci soliton $(\lambda + \xi f = 0)$ with $\sigma = 0$, then the manifold is Ricci-flat.*

Proof. From equation (5.2), if $\lambda + \xi f = 0$ and $\sigma = 0$, then

$$\mathcal{S}(Y, Z) = 0,$$

which implies the manifold is Ricci-flat. □

Theorem 5.5. *Let $(M, F, \varphi, \xi, \eta, g)$ be a Finsler LP-Sasakian manifold admitting a generalized η-Ricci soliton with potential vector field X such that $\eta(X) = 0$. Then the soliton reduces to a classical Ricci soliton:*

$$\mathcal{L}_X g + 2\mathcal{S} + 2\lambda g = 0.$$

Proof. If $\eta(X) = 0$, then $\eta \otimes \eta(X, Y) = 0$ for all Y. Hence, from (5.1), the last term in the soliton equation vanishes, reducing it to:

$$\mathcal{L}_X g + 2\mathcal{S} + 2\lambda g = 0,$$

which is the classical Ricci soliton condition. □

6. Examples

In this final section, we illustrate our theory with a model example of a Finsler LP-Sasakian manifold:

Let $M = \mathbb{R}^3 \setminus \{0\}$ with standard coordinates (x, y, z). Define the Finsler structure $F : TM \to [0, \infty)$ by the Randers-type metric:

$$F(x, y; y^i) = \sqrt{(y^1)^2 + (y^2)^2 + (y^3)^2} + \beta_i(x) y^i, \tag{6.1}$$

where $\beta = \eta = dz$ is a 1-form on M satisfying $\|\beta\|_g < 1$, ensuring strong convexity of F [5]. Let us define the structure tensors as:

$$\xi = \frac{\partial}{\partial z}, \quad \eta = dz, \tag{6.2}$$

$$\varphi\left(\frac{\partial}{\partial x}\right) = \frac{\partial}{\partial y}, \quad \varphi\left(\frac{\partial}{\partial y}\right) = \frac{\partial}{\partial x}, \quad \varphi\left(\frac{\partial}{\partial z}\right) = 0. \tag{6.3}$$

These structures satisfy the relations:

$$\eta(\xi) = 1, \quad \varphi^2 = I - \eta \otimes \xi, \quad \eta \circ \varphi = 0, \quad \varphi\xi = 0,$$

and can be shown to fulfill the Finsler LP-condition (3.7) with respect to the Chern connection. Therefore, $(M, F, \varphi, \xi, \eta)$ defines a Finsler LP-Sasakian manifold.

To study a soliton structure on this model, we consider the vector field $X = f(z)\,\xi$ and compute:

$$\mathcal{L}_X g = f'(z)\, g, \quad \text{implying} \quad (\mathcal{L}_X g)(Y, Z) = f'(z)\, g(Y, Z).$$

This leads to a generalized η-Ricci soliton of the form:

$$\mathcal{S} + (\lambda + f')g + \sigma\,\eta \otimes \eta = 0.$$

Hence, this model realizes the theoretical framework developed in Sections 5.

7. Conclusion

In this work, we have proposed and studied a new class of Finsler manifolds equipped with LP-Sasakian-type structures, termed *Finsler LP-Sasakian manifolds*. By formulating appropriate analogues of the structure tensors (φ, ξ, η) and incorporating a compatible Finsler metric, we extended the geometric framework of LP-Sasakian manifolds to the direction-dependent setting of Finsler geometry.

We examined the structural equations governing the manifold and derived fundamental cur- vature identities, including conditions on the curvature tensor $\mathcal{R}$ and its interaction with the structure tensors. Several theorems and corollaries illustrated how classical identities from Lorentzian or contact geometry generalize naturally in the Finsler context.

Additionally, we introduced and analyzed generalized η-Ricci solitons on these manifolds. The soliton equation was studied under various assumptions on the potential vector field, including cases where it is collinear with the structure vector field ξ. Under these assumptions, we established that the Ricci tensor admits an η-Einstein form. Moreover, in the presence of $\mathcal{W}_2$- flatness or steady soliton conditions, the manifold exhibits simplified Ricci behavior, including Ricci-flatness in special cases.

The results presented here highlight the rich structure and potential of Finsler LP-Sasakian geometry, both from a theoretical and geometric flow perspective. The flexibility of Finsler geometry to model direction-dependent phenomena opens possibilities for further exploration, particularly in physics-inspired settings such as anisotropic spacetimes and relativistic field theories.

Future Work. Potential directions for further research include:

- Classification of Finsler LP-Sasakian manifolds under specific curvature conditions (e.g., constant φ-sectional curvature);
- Study of gradient generalized η-Ricci solitons and their variational interpretations;
- Analysis of these manifolds under other natural Finsler connections, such as the Berwald or Chern connections;
- Construction of explicit models using (α, β)-metrics, Randers metrics, or Kropina-type structures;
- Extension of the theory to generalized quasi-Einstein or Yamabe soliton settings.

References

1. M. I. Akbar and E. Kilisarslan, *Ricci solitons in paracontact metric manifolds with generalized connections*, Turk. J. Math., **47**(2023), 1021–1040.
2. M. Altunbaş, *Generalized η-Ricci solitons on f-Kenmotsu manifolds admitting a quarter symmetric metric connection*, J. Finsler Geom. Appl., **5**(1) (2024), 80–87.
3. M. Altunbaş, *Generalized η-Ricci solitons on LP-Sasakian manifolds admitting the general connection*, arXiv:2507.09954, 2025.
4. M. Anastasiei, *Finsler connections and contact structures*, Tensor (N.S.), **56** (1994), 23–30.
5. D. Bao, S. S. Chern, and Z. Shen, *An Introduction to Riemann-Finsler Geometry*, Springer, New York, 2000.
6. J. T. Cho and M. M. *Tripathi, Ricci solitons in contact geometry*, Differ. Geom. Appl., **29**(1) (2011), 108–116.
7. M. Matsumoto, *Foundations of Finsler Geometry and Special Finsler Spaces*, Kaiseisha Press, 1986.
8. K. Matsumoto, *On Lorentzian paracontact manifolds*, Bull. Yamagata Univ. Natur. Sci., **12**(1989), 151-156.
9. E. Minguzzi, *Finsler spacetimes and gravity*, Rep. Math. Phys., **73**(2014), 87–120.
10. R. S. Mishra and A. K. Pandey, *LP-Sasakian manifolds*, Indian J. Pure Appl. Math. **27**(1996), 245–250.
11. S. Pahan, *h-Almost conformal Ricci-Bourguignon soliton on generalized Sasakian space form with respect to quarter-symmetric metric connection*, J. Finsler Geom. Appl. **6**(1) (2025), 125-144.
12. E. Peyghan and A. Tayebi, *On paracontact Finsler structures*, Int. J. Geom. Methods Mod. Phys. **10**(2013), 1350024.
13. J. Szilasi, R. L. Lovas, and D. C. Kertesz, *Connections, Sprays and Finsler Structures*, World Scientific, 2013.
14. S. Vacaru, *Finsler and Lagrange geometries in Einstein and string gravity*, Int. J. Geom. Methods Mod. Phys., **9**(2012), 1250041.

Received: 27.07.2025
Accepted: 07.11.2025