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On six dimensional Finsler spaces

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Abstract. The objective of this research paper is to comprehensively explore the main scalars within the context of the six dimensional Finsler space.This investigation leverages both $h-$ connection vectors and $v-$ connection vectors. Additionally we have introduced the $T-$ condition and $v-$ curvature tensor S_{hijk} and express them in an extended form in relation to scalars and tensors in terms of main scalars.

Keywords: Six- dimensional Finsler space, $h-$ connection vectors, $v-$ connection vectors, $T-$ condition.

1. Introduction

A theory of $n-$ dimensional Finsler space studied by Matsumoto and Miron [1, 2, 3] and is callled 'Miron Frame'. The Miron frame for six dimensional Finsler space is constructed by the unit vectors $(e^i_{1)}, e^i_{2)}, e^i_{3)}, e^i_{4)}, e^i_{5)}, e^i_{6)})$. The first vector $e^i_{1)}$ is the normalized supporting element l^i and second vector $e^i_{2)}$ is the normalized torsion vector $m^i = \frac{C^i}{C}$ where C is the length of torsion vector C^i. The third vector $e^i_{3)} = n^i$, fourth vector $e^i_{4)} = p^i$, fifth vector $e^i_{5)} = q^i$ and the sixth vector $e^i_{6)} = r^i$ are constructed by the Matsumoto and Miron [2]. Many investigators such as M. K. Gupta, P. N. Pandey [4] and S. K. Tiwari, Anamika Rai [7] have studied on four and five-dimensional Finsler spaces. P.

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K. Dwivedi, S. C. Rastogi and A. K. Dwivedi [6] have initially studied Finsler space in terms of scalars.
The purpose of the present investigation is to study the six- dimensional Finsler spaces satisfying T- conditions and V- curvature tensors in terms of main scalars with h- and v- connection vectors.

2. **Preliminaries**

In view of [2] $l_i = \dot{\partial}_i L$, $g_{ij} = \frac{1}{2}\dot{\partial} l_i \dot{\partial}_j L^2$, $h_{ij} = L\dot{\partial} l_i \dot{\partial}_j L$ and $C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij}$ respectvely. The $h-$ and $v-$ covariant derivatives of a covariant vector $X_i(x,y)$ with respect to Cartan connection are given by

$$X_{i/j} = \partial_j X_i - (\dot{\partial}_h X_i) G^h_j - F^r_{ij} X_r$$

and

$$X_i/j = \dot{\partial}_j X_i - C^r_{ij} X_r$$

With respect to frame [2], the scalar components of an arbitrary tensor T^i_j are defined by

$$T_{\alpha\beta} = T^i_j e_{\alpha)i} e^j_{\beta)} \tag{2.1}$$

From this we get

$$T^i_j = T_{\alpha\beta} e^i_{\alpha)} e_{\beta)j} \tag{2.2}$$

where summation convension is also applied to greek indices. The scalar components of the metric tensor g_{ij} are $g_{\alpha\beta}$ [4] and Angular metric tensor h_{ij} [7] of F^6 are given by

$$\begin{cases} g_{ij} = l_i l_j + m_i m_j + n_i n_j + p_i p_j + q_i q_j + r_i r_j, \\ h_{ij} = m_i m_j + n_i n_j + p_i p_j + q_i q_j + r_i r_j \end{cases} \tag{2.3}$$

Let $H_{\alpha)\beta\gamma}$ and $V_{\alpha)\beta\gamma}$ be scalar components of the $h-$ and $v-$ covariant derivative $e^i_{\alpha)/j}$ and $e^i_{\alpha)}/j$ respectively of the vector $e^i_{\alpha)}$, then

$$e^i_{\alpha)/j} = H_{\alpha)\beta\gamma} e^i_{\beta)} e_{\gamma)j} \& L e^i_{\alpha)\text{J}} = V_{\alpha)\beta\gamma} e^i_{\beta}) e_{\gamma)j} \tag{2.4}$$

The $h-$ covariant derivative for tensor X_i is given in above equation. $H_{\alpha)\beta\gamma}$ and $V_{\alpha)\beta\gamma}$, are called $h-$ and $v-$connection scalars respectively and positively homogeneous of degree $'0'$ in y. Orthogonality of the Miron Frame[2] yields

$$H_{\alpha)\beta\gamma} = -H_{\beta)\alpha\gamma} \text{and} V_{\alpha)\beta\gamma} = -V_{\beta)\alpha\gamma} \tag{2.5}$$

Also, we have $H_{1)\beta\gamma} = 0$ and $V_{1)\beta\gamma} = \delta_{\beta\gamma} - \delta_{1\beta}\delta_{1\gamma}$

Now we define h-connection and v- connection vector fields:

$$\begin{aligned}
H_{2)3\beta}e^{j}_{\beta)} &= h_j = -H_{3)2\beta}e^{j}_{\beta)}, H_{3)4\beta}e^{j}_{\beta)} = j_j = -H_{4)3\beta}e^{j}_{\beta)},\\
H_{4)2\beta}e^{j}_{\beta)} &= k_j = -H_{2)4\beta}e^{j}_{\beta)}, H_{5)2\beta}e^{j}_{\beta)} = p_j = -H_{2)5\beta}e^{j}_{\beta)},\\
H_{5)3\beta}e^{j}_{\beta)} &= q_j = -H_{3)5\beta}e^{j}_{\beta)}, H_{5)4\beta}e^{j}_{\beta)} = h^{'}_j = -H_{4)5\beta}e^{j}_{\beta)},\\
H_{6)2\beta}e^{j}_{\beta)} &= j^{'}_j = -H_{2)6\beta}e^{j}_{\beta)}, H_{6)3\beta}e^{j}_{\beta)} = k^{'}_j = -H_{3)6\beta}e^{j}_{\beta)},\\
H_{6)4\beta}e^{j}_{\beta)} &= p^{'}_j = -H_{4)6\beta}e^{j}_{\beta)}, H_{6)5\beta}e^{j}_{\beta)} = q^{'}_j = -H_{5)6\beta}e^{j}_{\beta)},
\end{aligned} \tag{2.6}$$

and

$$\begin{aligned}
V_{2)3\beta}e^{j}_{\beta)} &= u_j = -V_{3)2\beta}e^{j}_{\beta)}, V_{3)4\beta}e^{j}_{\beta)} = v_j = -V_{4)3\beta}e^{j}_{\beta)},\\
V_{4)2\beta}e^{j}_{\beta)} &= w_j = -V_{2)4\beta}e^{j}_{\beta)}, V_{5)2\beta}e^{j}_{\beta)} = r_j = -V_{2)5\beta}e^{j}_{\beta)},\\
V_{5)3\beta}e^{j}_{\beta)} &= s_j = -V_{3)5\beta}e^{j}_{\beta)}, V_{5)4\beta}e^{j}_{\beta)} = u^{'}_j = -V_{4)5\beta}e^{j}_{\beta)},\\
V_{6)2\beta}e^{j}_{\beta)} &= v^{'}_j = -V_{2)6\beta}e^{j}_{\beta)}, V_{6)3\beta}e^{j}_{\beta)} = w^{'}_j = -V_{3)6\beta}e^{j}_{\beta)},\\
V_{6)4\beta}e^{j}_{\beta)} &= r^{'}_j = -V_{4)6\beta}e^{j}_{\beta)}, V_{6)5\beta}e^{j}_{\beta)} = s^{'}_j = -V_{5)6\beta}e^{j}_{\beta)},
\end{aligned} \tag{2.7}$$

The vector fields h_j, j_j, k_j, p_j, q_j, $h^{'}_j$, $j^{'}_j$, $k^{'}_j$, $p^{'}_j$ and $q^{'}_j$ are called $h-$connection vectors and the vector fields u_j, v_j, w_j, r_j, s_j, $u^{'}_j$, $v^{'}_j$, $w^{'}_j$, $r^{'}_j$, and $s^{'}_j$ are called $v-$connection vectors. The scalars$H_{2)3\beta}$, $H_{3)4\beta}$, $H_{4)2\beta}$, $H_{5)2\beta}$, $H_{5)3\beta}$, $H_{5)4\beta}$, $H_{6)2\beta}$, $H_{6)3\beta}$, $H_{6)4\beta}$, $H_{6)5\beta}$ and $V_{2)3\beta}$, $V_{3)4\beta}$, $V_{4)2\beta}$, $V_{5)2\beta}$, $V_{5)3\beta}$, $V_{5)4\beta}$, $V_{6)2\beta}$, $V_{6)3\beta}$, $V_{6)4\beta}$, $V_{6)5\beta}$ are considerd as the scalar components h_j, j_j, k_j, p_j, q_j, $h^{'}_j$,$j^{'}_j$, $k^{'}_j$, $p^{'}_j$, $q^{'}_j$ and u_j, v_j, w_j, r_j, s_j, $u^{'}_j$, $v^{'}_j$, $w^{'}_j$, $r^{'}_j$, $s^{'}_j$ of the $h-$ and $v-$ connection vectors respectively with respect to the orthonormal frame. Let $C_{\alpha\beta\gamma}$ are the scalar components of LC_{ijk}, then

$$LC_{ijk} = C_{\alpha\beta\gamma}e_{\alpha)i}e_{\beta)j}e_{\gamma)k} \tag{2.8}$$

Then, we have from [6]

$$LC_{ijk} = C_{(222)}m_im_jm_k + C_{(233)}n_{(1)|}n_{(1)j}n_{(1)k} + C_{(244)}n_{(2)|}n_{(2)j}n_{(2)k} +$$
$$C_{(255)}n_{(3)|}n_{(3)j}n_{(3)k} + C_{(266)}n_{(4)|}n_{(4)j}n_{(4)k}$$
$$+ \sum_{(ijk)}[C_{(322)}m_im_jn_{(1)k} + C_{(333)}m_im_jn_{(2)k} +$$
$$C_{(344)}m_im_jn_{(3)k} + C_{(355)}m_im_jn_{(4)k} + C_{(366)}n_{(1)|}n_{(1)j}m_k +$$
$$C_{(422)}n_{(1)|}n_{(1)j}n_{(2)k} + C_{(433)}n_{(1)|}n_{(1)j}n_{(3)k}$$
$$+ C_{(444)}n_{(1)|}n_{(1)j}n_{(4)k} + C_{(455)}n_{(2)|}n_{(2)j}m_k +$$
$$C_{(466)}n_{(2)|}n_{(2)j}n_{(1)k} + C_{(522)}n_{(2)|}n_{(2)j}n_{(3)k}$$
$$+ C_{(533)}n_{(2)|}n_{(2)j}n_{(4)k} + C_{(544)}n_{(3)|}n_{(3)j}m_k +$$
$$C_{(555)}n_{(3)|}n_{(3)j}n_{(1)k} + C_{(566)}n_{(3)|}n_{(3)j}n_{(2)k} +$$
$$C_{(622)}n_{(3)|}n_{(3)j}n_{(4)k} + C_{(633)}n_{(4)|}n_{(4)j}m_k + C_{(644)}n_{(4)|}n_{(4)j}n_{(1)k}$$
$$+ C_{(655)}n_{(4)|}n_{(4)j}n_{(2)k} + C_{(666)}n_{(4)|}n_{(4)j}n_{(3)k} + C_{(234)}m_i(n_{(1)j}n_{(2)k} + n_{(1)k}n_{(4)j}) +$$
$$C_{(235)}m_i(n_{(1)j}n_{(3)k} + n_{(1)k}n_{(3)j}) + C_{(236)}m_i(n_{(1)j}n_{(4)k} + n_{(1)k}n_{(4)j}) +$$
$$C_{(245)}m_i(n_{(2)j}n_{(3)k} + n_{(2)k}n_{(3)j}) + C_{(246)}m_i(n_{(2)j}n_{(4)k} + n_{(2)k}n_{(4)j}) +$$
$$C_{(256)}m_i(n_{(3)j}n_{(4)k} + n_{(3)k}n_{(4)j}) + C_{(345)}n_{(1)|}(n_{(2)j}n_{(3)k} + n_{(2)k}n_{(3)j}) +$$
$$C_{(346)}n_{(1)i}(n_{(2)j}n_{(4)k} + n_{(2)k}n_{(4)j}) + C_{(356)}n_{(1)|}(n_{(3)j}n_{(4)k} + n_{(3)k}n_{(4)j}) +$$
$$C_{(456)}n_{(2)i}(n_{(3)j}n_{(4)k} + n_{(3)k}n_{(4)j})] \tag{2.9}$$

The main scalars of six dimensional Finsler space are given by [6]

$$C_{(222)} = A, C_{(233)} = B, C_{(244)} = C, C_{(255)} = D, C_{(266)} = E, C_{(333)} = F,$$
$$C_{(344)} = G, C_{(355)} = H, C_{(366)} = I, C_{(433)} = J, C_{(444)} = K, C_{(455)} = L,$$
$$C_{(466)} = M, C_{(533)} = N, C_{(544)} = A^{'}, C_{(555)} = B^{'}, C_{(566)} = C^{'},$$
$$C_{(633)} = D^{'}, C_{(644)} = E^{'}, C_{(655)} = F^{'}, C_{(666)} = G^{'}, C_{(234)} = H^{'},$$
$$C_{(235)} = I^{'}, C_{(236)} = J^{'}, C_{(245)} = K^{'}, C_{(246)} = L^{'}, C_{(256)} = M^{'}, C_{(345)} = N^{'},$$
$$C_{(346)} = A^{''}, C_{(356)} = B^{''}, C_{(456)} = C^{''},$$
$$\text{We also have}, A + J + N + D^{'}, +H^{'} = LC, C_{(223)} = -(B + A^{'} + E^{'} + I^{'}),$$
$$C_{(224)} = -(C + G + F^{'} + J^{'}), C_{(225)} = -(D + H + L + K^{'}),$$
$$C_{(226)} = -(E + I + M + C^{'}), \tag{2.10}$$

where LC is called the unified main scalar.
Taking $h-$ covariant derivative of vector field T^i_j with respect to k, we get

$$T^i_{j/k} = (\delta_k T_{\alpha\beta})e^i_{\alpha)}e_{\beta)j} + T_{\alpha\beta}e^i_{\alpha)/k}e_{\beta)j} + T_{\alpha\beta}e^i_{\alpha)}e_{\beta)j/k} \tag{2.11}$$

where $\delta_k = \partial_k G^r_k \dot{\partial}_r$. If $T_{\alpha\beta,\gamma}$ are the scalar components of $T^i_{j/k}$, i.e

$$T^i_{j/k} = T_{\alpha\beta,\gamma} e^i_{\alpha)} e_{\beta)j}, \tag{2.12}$$

then we obtain

$$T_{\alpha\beta,\gamma} = (\delta_k T_{\alpha\beta}) e^k_{\gamma)} + T_{\mu\beta} H_{\mu)\alpha\gamma} + T_{\alpha\mu} H_{\mu)\beta\gamma} \tag{2.13}$$

Similarly, if $T_{\alpha\beta;\gamma}$ are the scalar components of LT^i_j/k, i.e,

$$LT^i_j/k = T_{\alpha\beta;\gamma} e^i_{\alpha)} e_{\beta)j} e_{\gamma)k} \tag{2.14}$$

then we get

$$T_{\alpha\beta;\gamma} = L(\dot{\partial}_k T_{\alpha\beta}) e^k_{\gamma)} + T_{\mu\beta} V_{\mu)\alpha\gamma} + T_{\alpha\mu)} V_{\mu)\beta\gamma} \tag{2.15}$$

Equation (2.13) and (2.14) are called h- and v- scalar derivatives of the component $T_{\alpha\beta}$ of T^i_j.

3. T-Condition

M. Matsumoto, H. Shimada [5] and M. K. Gupta, P. N. Pandey [4] have been studied T- tensor defined as

$$T_{hijk} = LC_{ij/k} + C_{hij} l_k + C_{hik} l_j + C_{hkj} l_i + C_{kij} l_h \tag{3.1}$$

It is completely symmetric in its indices. A Finsler space is said to be satisfy $T-$ condition if the $T-$tensor T_{hijk} vanishes identically.
From (2.8) and (2.14), it follows that

$$L^2 C_{hij/k} + LC_{hij} l_k = C_{\alpha\beta\gamma;\delta} e_{\alpha)h} e_{\beta)i} e_{\gamma)j} e_{\delta)k}, \tag{3.2}$$

which implies

$$L^2 C_{hij/k} = (C_{\alpha\beta\gamma;\delta} - C_{\alpha\beta\gamma} \delta_{/\delta)} e_{\alpha)h} e_{\beta)i} e_{\gamma)j} e_{\delta)k} \tag{3.3}$$

Therefore the scalar components $T_{\alpha\beta\gamma\delta}$ of LT_{hijk} are given by

$$T_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma;\delta} + \delta_{/\alpha} C_{\beta\gamma\delta} + \delta_{/\beta} C_{\alpha\gamma\delta} + \delta_{/\gamma} C_{\alpha\beta\delta} \tag{3.4}$$

From $T_{hijk} l^k = 0$, we have $T_{\alpha\beta\gamma/\delta} = 0$. Thus the surviving components $T_{\alpha\beta\gamma\delta}$ are given by

$$T_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma;\delta}; \alpha, \beta, \gamma, \delta = 2, 3, 4, 5, 6 \tag{3.5}$$

If $T_{\alpha\beta;\gamma}$ the scalar components of LT^i_j/k, i.e

$$LT^i_j/k = T_{\alpha\beta;\gamma} e^i_{\alpha)} e_{\beta)j} e_{\gamma)k}, \tag{3.6}$$

then

$$T_{\alpha\beta;\gamma} = L(\dot{\partial}_k T_{\alpha\beta} e^k_{\gamma)} + T_{\mu\beta} V_{\mu)\alpha\beta} + T_{\alpha\mu} V_{\mu)\beta\gamma} \tag{3.7}$$

Similarly, taking $h-$ covariant differentiation of T^i_j with respect to k we get

$$T^i_{j/k} = (\delta_k T_{\alpha\beta} e^i_{\alpha)} e_{\beta)j} + T_{\alpha\beta} e^i_{\alpha)/k} e_{\beta)j} + T_{\alpha\beta} e^i_{\alpha)} e_{\beta)j/k} \tag{3.8}$$

where $\delta_k = \partial_k G^r_k \dot{\partial}_r$. If $T_{\alpha\beta,\gamma}$ are scalar components of $T^i_{j/k}$, i.e

$$T^i_{j/k} = T_{\alpha\beta,\gamma} e^i_{\alpha)} e_{\beta)j}, \tag{3.9}$$

then we obtain

$$T_{\alpha\beta,\gamma} = (\partial_k T_{\alpha\beta}) e^k_{\gamma)} + T_{\mu\beta} H_{\mu)\alpha\gamma} + T_{\alpha\mu} H_{\mu)\beta\gamma} \tag{3.10}$$

The scalar components $T_{\alpha\beta;\gamma}$ and $T_{\alpha\beta,\gamma}$ are respectively called $v-$ and $h-$ scalar derivatives of scalars components $T_{\alpha\beta}$ of T. By using (2.13) the explicit form of $C_{\alpha\beta\gamma,\delta}$ are obtained as:

$$\begin{cases}
C_{222,\delta} = A_{,\delta} + 3(B + A^{'} + E^{'} + I^{'})h_j + 3(C + G + F^{'} + J^{'})k_j + \\
3(D + H + L + K^{'})p_j + 3(E + I + M + C^{'})j_j, \\
C_{223,\delta} = -(B + A^{'} + E^{'} + I^{'})_{,\delta} - 2H^{'}k_j + (C + G + F^{'} + J^{'})q_j - 2J^{'}h_j - \\
2I^{'}p_j + (D + H + L + K^{'})q_j - 2J^{'}j^{'}_j + (E + I + M + C^{'})k^{'}_j, \\
C_{224,\delta} = -(C + G + F^{'} + J^{'})_{,\delta} - 2H^{'}h_j + (B + A^{'} + \\
E^{'} + I^{'})j_j - \\
2Ck_j - 2K^{'}p_j + (D + H + L + K^{'})h^{'}_j - 2L^{'}j^{'}_j + (E + I + M + C^{'})p^{'}_j, \\
C_{225,\delta} = -(D + H + L + K^{'})_{,\delta} - 2I^{'}h_j - (B + A^{'} + E^{'} + I^{'})q_j - 2K^{'}k_j - 2Dp_j - (C + G + F^{'} + \\
J^{'})h^{'}_j - 2M^{'}j^{'}_j + (E + I + M + C^{'})q^{'}_j, \\
C_{226,\delta} = -(E + I + M + C^{'})_{,\delta} - 2J^{'}h_j - (B + A^{'} + E^{'} + I^{'})k^{'}_j - 2L^{'}k_j - 2M^{'}p_j - (C + G + F^{'} + \\
J^{'})p^{'}_j - 2Ej^{'}_j - (D + H + L + K^{'})q^{'}_j, \\
C_{233,\delta} = B_{,\delta} - Fh_j - 2H^{'}j_j - Jk_j - Np_j - 2I^{'}q_j - D^{'}j^{'}_j - 2J^{'}k^{'}_j, \\
C_{244,\delta} = C_{,\delta} - Gh_j - 2H^{'}j_j - Kk_j - A^{'}p_j - 2K^{'}h^{'}_j - E^{'}j^{'}_j - 2L^{'}p^{'}_j, \\
C_{255,\delta} = D_{,\delta} - Hh_j - 2I^{'}q_j - Lk_j - B^{'}p_j - 2K^{'}h^{'}_j - F^{'}j^{'}_j - 2M^{'}p^{'}_j, \\
C_{266,\delta} = E_{,\delta} - Ih_j + 2J^{'}k^{'}_j - Mk_j - C^{'}p_j + 2M^{'}q^{'}_j - G^{'}j^{'}_j + 2L^{'}p^{'}_j, \\
C_{333,\delta} = F_{,\delta} - 3Jj_j - 3Hq_j - 3D^{'}p^{'}_j, \\
C_{344,\delta} = G_{,\delta} + 2Jj_j - Kj_j - 3A^{'}q_j - 2N^{'}h^{'}_j - E^{'}j^{'}_j - 2A^{''}p^{'}_j, \\
C_{355,\delta} = H_{,\delta} + 2Nq_j - Lj_j - B^{'}q_j - 2N^{'}h^{'}_j - F^{'}k^{'}_j - 2B^{''}q^{'}_j, \\
C_{366,\delta} = I_{,\delta} + 2D^{'}k^{'}_j - Mj_j - B^{'}q_j - 2A^{''}p^{'}_j - C^{'}q^{'}_j + 2B^{''}q^{'}_j + G^{'}q^{'}_j, \\
C_{433,\delta} = J_{,\delta} - Nh^{'}_j + Fj_j - 2Gj_j - 2N^{'}q_j - 2A^{''}k^{'}_j - D^{'}p^{'}_j, \\
C_{444,\delta} = K_{,\delta} + 3Gj_j - 3A^{'}h^{'}_j - 3E^{'}p^{'}_j, \\
C_{455,\delta} = L_{,\delta} + Hj_j - 2N^{'}q_j - 2A^{'}h^{'}_j - B^{'}p^{'}_j - F^{'}k^{'}_j - 2C^{"}q^{'}_j, \\
C_{466,\delta} = M_{,\delta} + Ij_j - G^{'}p^{'}_j + 2A^{''}k^{'}_j + 2E^{'}p^{'}_j - C^{'}h^{'}_j + 2C^{''}q^{'}_j, \\
C_{533,\delta} = N_{,\delta} + Fq_j - Jh^{'}_j - 2N^{'}j_j - 2Hq_j - D^{'}q^{'}_j - 2B^{''}k^{'}_j, \\
C_{544,\delta} = A^{'}_{,\delta} + Gq_j - 2N^{'}j_j + Kh^{'}_j - 2Lh^{'}_j - E^{'}q^{'}_j - 2C^{''}p^{'}_j, \\
C_{555,\delta} = B^{'}_{,\delta} + 3Hq_j - 3Lh^{'}_j - 3F^{'}q^{'}_j,
\end{cases} \tag{3.11}$$

$$\begin{cases}
C_{566,\delta}=C'_{,\delta}+Iq_j-2B''k'_j+Mh'_j-G'q'_j-2F''q'_j+2C''p'_j,\\
C_{633,\delta}=D'_{,\delta}+Fk'_j+Jp'_j-2A''j_j+Nq'_j-2B''q_j-2Ik'_j,\\
C_{644,\delta}=E'_{,\delta}+Gk'_j+Kp'_j+2A''j_j+A'q'_j-2C''h'_j-2Mp'_j,\\
C_{655,\delta}=F'_{,\delta}+Hk'_j+Lp'_j+2B''q_j+B'q'_j+2C''h'_j-2C'q'_j,\\
C_{666,\delta}=G'_{,\delta}+3Ik'_j+3Mp'_j+3C'q'_j,\\
C_{234,\delta}=H'_{,\delta}-Jh_j+Bj_j-Gk_j-Cj_j-N'p_j-K'q_j+Ih'_j-2A''j'_j-L'k'_j-J'q'_j,\\
C_{235,\delta}=I'_{,\delta}-Nh_j+Bq_j-Hp_j-K'j_j-N'k_j-Dq_j+H'h'_j-B''j'_j-M'k'_j-J'q'_j,\\
C_{236,\delta}=J'_{,\delta}-D'h_j+Bk'_j-A''k_j-L'j_j-H'p'_j-B''p_j-M'q_j-I'q'_j-Ij'_j-Ek'_j,\\
C_{245,\delta}=K'_{,\delta}-N'h_j-A'k_j-I'j_j-Ch'_j-Lp_j+H'q_j-Dh'_j-C''j'_j-M'p'_j-L'q'_j,\\
C_{246,\delta}=L'_{,\delta}-A''h_j+H'k'_j+J'j_j+Cp'_j-E'k_j-C''p_j-M'h'_j-Mj'_j-Ep'_j+K'q'_j,\\
C_{256,\delta}=M'_{,\delta}-B''h_j+I'k'_j+J'q_j+K'p'_j-C''k_j-L'h'_j-C'j'_j+Dq'_j-Eq'_j-F'p_j,\\
C_{345,\delta}=N'_{,\delta}+Nj_j-A'j_j+Jq_j+Gh'_j-Lq_j-Hh'_j-C''k'_j+B''q'_j-A''q'_j,\\
C_{346,\delta}=A''_{,\delta}+D'j_j+Jk'_j-E'j_j-C''q_j+Gp'_j-B''h'_j-Mk'_j+N'q'_j-Ip'_j,\\
C_{356,\delta}=B''_{,\delta}+D'q_j+Nk'_j-A''j_j-B''q_j+N'p'_j+A''h'_j-Ik'_j+Hq'_j-Iq'_j,\\
C_{456,\delta}=C''_{,\delta}+N'k'_j+B''j_j+A''q_j+A'p'_j+E'h'_j-F'h'_j+Lq'_j-Mq'_j-C'p'_j,\\
C_{1\beta\gamma,\delta}=0
\end{cases}$$

where $A_{,\delta}=(\partial_k A)e^k_{\gamma)}$. Hence, we have the following result.

Theorem 3.1. *In a six dimensional Finsler space the h- covariant derivatives in terms of main scalars satisfying T- condition is given by equation* (3.11).

Similarly,using (2.15) the explicit form of $C_{\alpha\beta\gamma;\delta}$ are obtained as follows:

$$\begin{cases}
C_{222;\delta}=A_{;\delta}+3(B+A'+E'+I')u_j+3(C+G+F'+J')w_j+\\
3(D+H+L+K')r_j+3(E+I+M+C')v_j,\\
C_{223;\delta}=-(B+A'+E'+I')_{;\delta}-2H'w_j+(C+G+F'+J')s_j-2J'u_j-2I'r_j+\\
(D+H+L+K')s_j-2J'v'_j+(E+I+M+C')w'_j,\\
C_{224;\delta}=-(C+G+F'+J')_{;\delta}-2H'u_j+(B+A'+E'+I')v_j-2Cw_j-2K'r_j+\\
(D+H+L+K')u'_j-2L'v'_j+(E+I+M+C')r'_j,\\
C_{225;\delta}=-(D+H+L+K')_{;\delta}-2I'u_j-(B+A'+E'+I')s_j-2K'w_j-2Dr_j-\\
(C+G+F'+J')u'_j-2M'v'_j+(E+I+M+C')s'_j,\\
C_{226;\delta}=-(E+I+M+C')_{;\delta}-2J'u_j-(B+A'+E'+I')w'_j-2L'w_j-2M'r_j-\\
(C+G+F'+J')r'_j-2Ev'_j-(D+H+L+K')s'_j,\\
C_{233;\delta}=B_{;\delta}-Fu_j-2H'v_j-Jw_j-Nr_j-2I's_j-D'v'_j-2J'w'_j,\\
C_{244;\delta}=C_{;\delta}-Gu_j-2H'v_j-Kw_j-A'r_j-2K'u'_j-E'v'_j-2L'r'_j,
\end{cases} \qquad (3.12)$$

$$
\begin{cases}
C_{255;\delta} = D_{;\delta} - Hu_j - 2I^{'}s_j - Lw_j - B^{'}r_j - 2K^{'}u^{'}_j - F^{'}v^{'}_j - 2M^{'}r^{'}_j,\\
C_{266;\delta} = E_{;\delta} - Iu_j + 2J^{'}w^{'}_j - Mw_j - C^{'}r_j + 2M^{'}s^{'}_j - G^{'}v^{'}_j + 2L^{'}r^{'}_j,\\
C_{333;\delta} = F_{;\delta} - 3Jv_j - 3Hs_j - 3D^{'}r^{'}_j,\\
C_{344;\delta} = G_{;\delta} + 2Jv_j - Kv_j - 3A^{'}s_j - 2N^{'}u^{'}_j - E^{'}v^{'}_j - 2A^{''}r^{'}_j,\\
C_{355;\delta} = H_{;\delta} + 2Ns_j - Lv_j - B^{'}s_j - 2N^{'}u^{'}_j - F^{'}w^{'}_j - 2B^{''}s^{'}_j,\\
C_{366;\delta} = I_{;\delta} + 2D^{'}w^{'}_j - Mv_j - B^{'}s_j - 2A^{''}r^{'}_j - C^{'}s^{'}_j + 2B^{''}s^{'}_j + G^{'}s^{'}_j,\\
C_{433;\delta} = J_{;\delta} - Nu^{'}_j + Fv_j - 2Gv_j - 2N^{'}s_j - 2A^{''}w^{'}_j - D^{'}r^{'}_j,\\
C_{444;\delta} = K_{;\delta} + 3Gv_j - 3A^{'}u^{'}_j - 3E^{'}r^{'}_j,\\
C_{455;\delta} = L_{;\delta} + Hv_j - 2N^{'}s_j - 2A^{'}u^{'}_j - B^{'}r^{'}_j - F^{'}w^{'}_j - 2C^{"}s^{'}_j,\\
C_{466;\delta} = M_{;\delta} + Iv_j - G^{'}r^{'}_j + 2A^{''}w^{'}_j + 2E^{'}r^{'}_j - C^{'}u^{'}_j + 2C^{''}s^{'}_j,\\
C_{533;\delta} = N_{;\delta} + Fs_j - Ju^{'}_j - 2N^{'}v_j - 2Hs_j - D^{'}s^{'}_j - 2B^{''}w^{'}_j,\\
C_{544;\delta} = A^{'}_{;\delta} + Gs_j - 2N^{'}v_j + Ku^{'}_j - 2Lu^{'}_j - E^{'}s^{'}_j - 2C^{''}r^{'}_j,\\
C_{555;\delta} = B^{'}_{;\delta} + 3Hs_j - 3Lu^{'}_j - 3F^{'}s^{'}_j,\\
C_{566;\delta} = C^{'}_{;\delta} + Is_j - 2B^{''}w^{'}_j + Mu^{'}_j - G^{'}s^{'}_j - 2F^{''}s^{'}_j + 2C^{''}r^{'}_j,\\
C_{633;\delta} = D^{'}_{;\delta} + Fw^{'}_j + Jr^{'}_j - 2A^{''}v_j + Ns^{'}_j - 2B^{''}s_j - 2Iw^{'}_j,\\
C_{644;\delta} = E^{'}_{;\delta} + Gw^{'}_j + Kr^{'}_j + 2A^{''}v_j + A^{'}s^{'}_j - 2C^{''}u^{'}_j - 2Mr^{'}_j,\\
C_{655;\delta} = F^{'}_{;\delta} + Hw^{'}_j + Lr^{'}_j + 2B^{''}s_j + B^{'}s^{'}_j + 2C^{''}u^{'}_j - 2C^{'}s^{'}_j,\\
C_{666;\delta} = G^{'}_{;\delta} + 3Iw^{'}_j + 3Mr^{'}_j + 3C^{'}s^{'}_j,\\
C_{234;\delta} = H^{'}_{;\delta} - Ju_j + Bv_j - Gw_j - Cv_j - N^{'}r_j - K^{'}s_j + Iu^{'}_j - 2A^{''}v^{'}_j - L^{'}w^{'}_j - J^{'}s^{'}_j,\\
C_{235;\delta} = I^{'}_{;\delta} - Nu_j + Bs_j - Hr_j - K^{'}v_j - N^{'}w_j - Ds_j + H^{'}u^{'}_j - B^{''}v^{'}_j - M^{'}w^{'}_j - J^{'}s^{'}_j,\\
C_{236;\delta} = J^{'}_{;\delta} - D^{'}u_j + Bw^{'}_j - A^{''}w_j - L^{'}v_j - H^{'}r^{'}_j - B^{''}r_j - M^{'}s_j - I^{'}s^{'}_j - Iv^{'}_j - Ew^{'}_j,\\
C_{245;\delta} = K^{'}_{;\delta} - N^{'}u_j - A^{'}w_j - I^{'}v_j - Cu^{'}_j - Lr_j + H^{'}s_j - Du^{'}_j - C^{''}v^{'}_j - M^{'}r^{'}_j - L^{'}s^{'}_j,\\
C_{246;\delta} = L^{'}_{;\delta} - A^{''}u_j + H^{'}w^{'}_j + J^{'}v_j + Cr^{'}_j - E^{'}w_j - C^{''}r_j - M^{'}u^{'}_j - Mv^{'}_j - Er^{'}_j + K^{'}s^{'}_j,\\
C_{256;\delta} = M^{'}_{;\delta} - B^{''}u_j + I^{'}w^{'}_j + J^{'}s_j + K^{'}r^{'}_j - C^{''}w_j - L^{'}u^{'}_j - C^{'}v^{'}_j + Ds^{'}_j - Es^{'}_j - F^{'}r_j,\\
C_{345;\delta} = N^{'}_{;\delta} + Nv_j - A^{'}v_j + Js_j + Gu^{'}_j - Ls_j - Hu^{'}_j - C^{''}w^{'}_j + B^{''}s^{'}_j - A^{''}s^{'}_j,\\
C_{346;\delta} = A^{''}_{;\delta} + D^{'}v_j + Jw^{'}_j - E^{'}v_j - C^{''}s_j + Gr^{'}_j - B^{''}u^{'}_j - Mw^{'}_j + N^{'}s^{'}_j - Ir^{'}_j,\\
C_{356;\delta} = B^{''}_{;\delta} + D^{'}s_j + Nw^{'}_j - A^{''}v_j - B^{''}s_j + N^{'}r^{'}_j + A^{''}u^{'}_j - Iw^{'}_j + Hs^{'}_j - Is^{'}_j,\\
C_{456;\delta} = C^{''}_{;\delta} + N^{'}w^{'}_j + B^{''}v_j + A^{''}s_j + A^{'}r^{'}_j + E^{'}u^{'}_j - F^{'}u^{'}_j + Ls^{'}_j - Ms^{'}_j - C^{'}r^{'}_j,\\
C_{1\beta\gamma;\delta} = -C_{\beta\gamma\delta},
\end{cases}
$$

where $A_{;\delta} = (\dot{\partial}_k A)e^k_{\gamma)}$. Hence, we obtain the following result.

Theorem 3.2. *In a six dimensional Finsler space the v- covariant derivatives in terms of main scalars satisfying T- conditions is given by equation* (3.12).

4. V-Curvature Tensor

M. Matsumoto [3] defined $V-$ curvature tensor as

$$S_{hijk} = C^r_{hk}C_{ijr} - C^r_{hj}C_{ikr} \tag{4.1}$$

The scalar components of $S_{\alpha\beta\gamma\delta}$ of L^2S_{hijk} are given by [6, 7]

$$L^2S_{hijk} = S_{\alpha\beta\gamma\delta}e_{\alpha)h}e_{\alpha)i}e_{\gamma)j}e_{\delta)k} \tag{4.2}$$

In view of equation (4.1) and (4.2) we have obtained 54 independent components in terms of main scalars. Since S_{hijk} is skew symmetric in h and i as well as in j and k and S_{0ijk}=S_{hi0k}=0, the surviving independent components of $S_{\alpha\beta\gamma\delta}$ are 54, which are obtained as:

$$\begin{cases}
S_{2323} = C_{23\theta}C_{\theta 32} - C_{22\theta}C_{\theta 33} \\
= 2B^2 + A'^2 + E'^2 + H'^2 + 2I'^2 + J'^2 + 2BA' + 2BE' + 2BI' \\
+2A'E' + 2A'I' + 2E'I' - AB + FB + FA' + FE' + FI' + CJ \\
+JF' + JJ' + DN + HN + LN + NK' + ED' + ID' + MD' + C'D', \\
S_{2324} = C_{24\theta}C_{\theta 32} - C_{22\theta}C_{\theta 34} \\
= (C + G + F' + J')(B + A' + E' + I') - AH' \\
+H'B + (B + A' + E' + I')J + CH' + (C + G + I' + J')G \\
+K'I' + (D + H + L + K')N' + L'J' + (E + I + M + C')A'', \\
S_{2325} = C_{23\theta}C_{\theta 32} - C_{22\theta}C_{\theta 35} \\
= (D + H + L + K')(B + A' + E' + I') - AI' + I'B \\
+(B + A' + E' + I')N + K'H' + (C + G + F' + J')N' + DI' \\
+(D + H + L + K')H + M'J' + (E + I + M + C')A'', \\
S_{2326} = C_{26\theta}C_{\theta 32} - C_{22\theta}C_{\theta 36} \\
= (B + A' + E' + I')(E + I + M + C') - AJ' + BJ' \\
+(B + A' + E' + I')D' + L'H' + 9C + G + F' + J')A'' + M'I' \\
+9D + H + L + K')B'' + EJ' + (E + I + M + C')I, \\
S_{2334} = C_{24\theta}C_{\theta 33} - C_{23\theta}C_{\theta 34} \\
= -(C + G + F' + J')B + (B + A' + E' + I')H' + IH' - BJ \\
+CJ - GH' + NK' - I'N' + L'D' - J'A'', \\
S_{2335} = C_{25\theta}C_{\theta 33} - C_{23\theta}C_{\theta 35} \\
= -(D + H + L + K')B + (B + A' + E' + I')I' + FI' - BN \\
-JK' - HI' + DN + M'D' + L'D' - J'B'', \\
S_{2336} = C_{26\theta}C_{\theta 33} - C_{23\theta}C_{\theta 36} \\
= -(E + I + M + C')B + (B + A' + E' + I')J' + FJ' - BD' \\
+JL' - H'A'' + M'N - I'B'' + ED' - J'I,
\end{cases}$$

$$
\begin{cases}
S_{2424} = C_{24\theta}C_{\theta 42} - C_{22\theta}C_{\theta 44} \\
= (C+G+F'+J')^2 - AC + H'^2 + (B+A'+E'+I')G + C^2 \\
+(C+G+F'+J')K + K'^2 + (D+H+L+K')A' + L'^2 \\
+(E+I+M+C')E', \\
S_{2425} = C_{25\theta}C_{\theta 42} - C_{22\theta}C_{\theta 45} \\
= (D+H+L+K')(C+G+F'+J') - AK' + I'H' \\
+(B+A'+E'+I')N' + K'C + (C+G+F'+J')K' + DK' \\
+(D+H+L+K')L + M'L' + (E+I+M+C')C'', \\
S_{2426} = C_{26\theta}C_{\theta 42} - C_{22\theta}C_{\theta 46} \\
= (E+I+M+C')(C+G+F'+J') - AL' + J'H' \\
+(B+A'+E'+I')A'' + L'C + (C+G+F'+J')E' + M'K' \\
+(D+H+L+K')L' + EL' + (E+I+M+C')M, \\
S_{2345} = C_{25\theta}C_{\theta 34} - C_{24\theta}C_{\theta 35} \\
= -(D+H+L+K')H' + (C+G+F'+J')I' + JI' - NH' \\
+GK' - CN' + DN' - K'H + A''M' - L'B'', \\
S_{2346} = C_{26\theta}C_{\theta 33} - C_{24\theta}C_{\theta 36} \\
= (E+I+M+C')B + (C+G+F'+J')J' + FJ' \\
-J'D' - JL' - CA'' + NM' - B''K' + ED' - IL', \\
S_{2525} = C_{25\theta}C_{\theta 52} - C_{22\theta}C_{\theta 55} \\
= (D+H+L+K')^2 - AD + I'^2 + (B+A'+E'+I')H \\
+K'^2 + (C+G+F'+J')L + D^2 + (D+H+L+K')B' \\
+M'^2 + (E+I+M+C')F', \\
S_{2526} = C_{26\theta}C_{\theta 52} - C_{22\theta}C_{\theta 56} \\
= (D+H+L+K')^2 - AD + I'^2 + (B+A'+E'+I')H + K'^2 \\
+(C+G+F'+J')L + D^2 + (D+H+L+K')B' + M'^2 \\
+(E+I+M+C')F', \\
S_{2434} = C_{24\theta}C_{\theta 43} - C_{23\theta}C_{\theta 44} \\
= (D+H+L+K')H' + (B+A'+E'+I')C + JH' \\
-BG + CG - H'K + K'N' - A'I' + L'A'' - J'E', \\
S_{2435} = C_{25\theta}C_{\theta 43} - C_{23\theta}C_{\theta 45} \\
= -(D+H+L+K')H' + (B+A'+E'+I')K' + JI' \\
-BN' + K'G - H'A' + DN' - I'L + M'A'' - J'C'',
\end{cases}
$$

$$
\begin{cases}
S_{2436} = C_{26\theta}C_{\theta 43} - C_{23\theta}C_{\theta 46} \\
= -(E+I+M+C')H' + (B+A'+E'+I')L' + JJ' \\
-BA'' + L'G - H'E' + M'N' - I'C'' + EA'' - J'M, \\
S_{2445} = C_{25\theta}C_{\theta 44} - C_{24\theta}C_{\theta 45} \\
= -(D+H+L+K')C + (C+G+F'+J')K' + I'G \\
-H'N' + K'K - CA' + DA' - K'L + E'M' - L'C'', \\
S_{2446} = C_{26\theta}C_{\theta 44} - C_{24\theta}C_{\theta 46} \\
= -(E+I+M+C')C + (C+G+F'+J')L' + J'G \\
-H'A'' + L'K - CE' + M'A' - K'C'' + E'E - L'M, \\
S_{2456} = C_{26\theta}C_{\theta 45} - C_{25\theta}C_{\theta 46} \\
= -(E+I+M+C')K' + (D+H+L+K')L' + J'N' \\
-I'A''L'A' - K'E' + M'L - DC'' + C''E - M'M, \\
S_{2534} = C_{24\theta}C_{\theta 53} - C_{23\theta}C_{\theta 54} \\
= -(C+G+F'+J')I' + (B+A'+E'+I')K' + H'N \\
-BN' + CN' - H'A' + K'N - I'A' + L'M' - J'C'', \\
S_{2535} = C_{25\theta}C_{\theta 53} - C_{23\theta}C_{\theta 55} \\
= -(D+H+L+K')I' + (B+A'+E'+I')D + I'N \\
-BH - K'N' - H'L + DH - I'B' + M'B'', \\
S_{2536} = C_{26\theta}C_{\theta 53} - C_{23\theta}C_{\theta 56} \\
= -(E+I+M+C')I' + (B+A'+E'+I')M' + J'N \\
-BB' - L'N' - H'C'' + M'H - I'F' + EB'' - J'C', \\
S_{2545} = C_{25\theta}C_{\theta 54} - C_{24\theta}C_{\theta 55} \\
= -(D+H+L+K')K' + (C+G+F'+J')D + I'N' \\
-HH' + K'A' - CL + DL - K'B' + M'C'' - L'F', \\
S_{2546} = C_{26\theta}C_{\theta 54} - C_{24\theta}C_{\theta 56} \\
= -(E+I+M+C')K' + (C+G+F'+J')M' + J'N' \\
-H'B'' + L'A' - CC'' + M'L - K'F' + EC'' - L'C', \\
S_{2556} = C_{26\theta}C_{\theta 55} - C_{25\theta}C_{\theta 56} \\
= -(E+I+M+C')D + (D+H+L+K')M' + J'H \\
-I'B'' + L'L - K'C'' + L'B' - DF' + EF' - M'C', \\
S_{2626} = C_{26\theta}C_{\theta 62} - C_{22\theta}C_{\theta 66} \\
= (E+I+M+C')^2 - AE + J'^2 + (B+A'+E'+I')I \\
+L'^2 + (C+G+F'+J')M + M'^2 + (D+H+L+K')C' \\
+E^2 + (E+I+M+C')G', \\
S_{2634} = C_{24\theta}C_{\theta 63} - C_{23\theta}C_{\theta 64} \\
= -(C+G+I'+J')J' + (B+A'+E'+I')L' + H'D' \\
-BA'' + CA'' - H'E' + K'B'' - I'C'' + IL' - J'M, \\
S_{2635} = C_{25\theta}C_{\theta 63} - C_{23\theta}C_{\theta 65} \\
= -(D+H+L+K')J' + (B+A'+E'+I')M' + I'D' \\
-BB'' + K'A'' - H'C'' + DB'' - I'F' + IM' - J'C',
\end{cases}
\tag{4.3}
$$

$$
\begin{cases}
S_{2636} = C_{26\theta}C_{\theta 63} - C_{23\theta}C_{\theta 66} \\
= -(E+I+M+C')J' + (B+A'+E'+I')E + J'D' \\
-BI + L'A'' - MH' + M'B'' - I'C' + EI - J'G', \\
S_{2645} = C_{25\theta}C_{\theta 64} - C_{24\theta}C_{\theta 65} \\
= -(D+H+L+K')L' + (C+G+F'+J')M' + I'A'' \\
-H'B'' + K'E' - CC'' + DC'' - K'F' + M'M - L'C', \\
S_{2646} = C_{26\theta}C_{\theta 64} - C_{24\theta}C_{\theta 66} \\
= -(E+I+M+C')L' + (C+G+F'+J')E + J'A'' \\
-H'I + L'E' - CM + J'C'' - K'C' + EM - L'G', \\
S_{2656} = C_{26\theta}C_{\theta 65} - C_{25\theta}C_{\theta 66} \\
= -(E+I+M+C')M' + (D+H+L+K')E' + J'B'' \\
-I'I + L'C'' - K'M + M'F' - DC' + EC' - M'G', \\
S_{3434} = C_{34\theta}C_{\theta 43} - C_{33\theta}C_{\theta 44} \\
= H'^2 - BC + J^2 - FG + G^2 - JG + N'^2 - NA' + A''^2 - D'E', \\
S_{3446} = C_{36\theta}C_{\theta 44} - C_{34\theta}C_{\theta 46} \\
= J'C - H'L' + D'G - JA'' + A'K - GE' + A'B'' - N'C'' \\
+IE' - A''M, \\
S_{3456} = C_{36\theta}C_{\theta 45} - C_{35\theta}C_{\theta 46} \\
= J'K' - I'L' + D'N' - NA'' + A''A - N'E' + B''L \\
-HC'' + IC'' - B''M, \\
S_{3535} = C_{35\theta}C_{\theta 53} - C_{33\theta}C_{\theta 55} \\
= I'^2 - BD + N^2 - FH + N'^2 - JL + H^2 - NB' + B''^2 - D'F', \\
S_{3536} = C_{36\theta}C_{\theta 53} - C_{33\theta}C_{\theta 56} \\
= J'I' - BM' + D'N - FB'' + A''N' - JC'' + B''H \\
-NF' + IB'' - D'C', \\
S_{3435} = C_{35\theta}C_{\theta 43} - C_{33\theta}C_{\theta 45} \\
= I'H' - BK' + NJ - FN' + N'G - JA' + HN' \\
-NL + B''A'' - D'C'', \\
S_{3445} = C_{35\theta}C_{\theta 44} - C_{34\theta}C_{\theta 45} \\
= I'C - H'K' + NG - JN' + N'K - GA' + HA' \\
-N'L + B''E' - A''C'', \\
S_{3436} = C_{36\theta}C_{\theta 43} - C_{33\theta}C_{\theta 46} \\
= J'H' - BL' + D'J - FA'' + A''G - JE' + B''N' \\
-NC'' + IA'' - D'M, \\
S_{3545} = C_{35\theta}C_{\theta 54} - C_{34\theta}C_{\theta 55} \\
= I'k' - H'D + N'N - JH + A'N' - GL \\
-HL - N'B' + B''C'' - A''F', \\
S_{3546} = C_{36\theta}C_{\theta 54} - C_{34\theta}C_{\theta 56} \\
= J'H' - H'M' + D'J - JB'' + A''G - GC'' \\
+B''N' - N'F' + IA'' - A''C', \\
S_{3556} = C_{36\theta}C_{\theta 55} - C_{35\theta}C_{\theta 56} \\
= J'D - I'M' + D'H - NB'' + A''L - N'C'' \\
+B''B' - HF' + IF' - B''C',
\end{cases}
$$

$$
\begin{cases}
S_{3636} = C_{36\theta}C_{\theta 63} - C_{33\theta}C_{\theta 66} \\
= J'^2 - BE + D'^2 - FI + A''^2 - JM + B'^2 - NC' + I^2 - D'G', \\
S_{3645} = C_{35\theta}C_{\theta 64} - C_{34\theta}C_{\theta 65} \\
= I'L' - H'M' + NA'' - JB'' + N'E' - GC'' \\
+HC'' - N'F' + B''M - A''C', \\
S_{3646} = C_{36\theta}C_{\theta 64} - C_{34\theta}C_{\theta 66} \\
= J'L' - H'E + D'A'' - JE + A''E' - GM + B''C'' - N'C' + IM - A''G', \\
S_{3656} = C_{36\theta}C_{\theta 65} - C_{35\theta}C_{\theta 66} \\
= J'M' - I'E + D'B'' - NI + A''B'' - N'M \\
+B''F' - HC' + IC' - B''G', \\
S_{4545} = C_{45\theta}C_{\theta 54} - C_{44\theta}C_{\theta 55} \\
= K'^2 - CD + N'^2 - GH + A'^2 - KL + L^2 - A'B' + C''^2 - E'F', \\
S_{4546} = C_{46\theta}C_{\theta 54} - C_{44\theta}C_{\theta 56} \\
= L'K' - CM' + N'A'' - GB'' + C''L - A'F' + MC'' - E'C', \\
S_{4556} = C_{46\theta}C_{\theta 55} - C_{45\theta}C_{\theta 56} \\
= L'D - K'M' + A''H - N'B'' + E'L - A'C'' \\
+C''B' - LF' + MF' - C''C', \\
S_{4646} = C_{46\theta}C_{\theta 64} - C_{44\theta}C_{\theta 66} \\
= L'^2 - CE + A''^2 - GI + E'^2 - KM + C''^2 - A'C' + M^2 - E'G', \\
S_{4656} = C_{46\theta}C_{\theta 65} - C_{45\theta}C_{\theta 66} \\
= L'M' - K'E + A''B'' - N'I + E'C'' - A'M \\
+C''F' - LC' + MC' - C''G' \\
S_{5656} = C_{56\theta}C_{\theta 65} - C_{55\theta}C_{\theta 66} \\
= M'^2 - DE + B''^2 - HI + C''^2 - LM \\
+F'^2 - B'C' + C'^2 - F'G',
\end{cases}
$$

Theorem 4.1. *In a six- dimensional Finsler space the V-curvature tensor in terms of main scalars are given by equation* (4.3).

Conclusion

This research paper provides fundamental insights into the relationship between tensors and scalars of Finsler spaces, highlighting the intricate nature of this mathematical framework. Overall it offers a detailed exploration of tensors and their derivative in Finsler spaces.

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